12.3 ODDS

Mathematical Concepts

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Definition:

- The **odds against** an event is a ratio of the probability that the event will fail to occur (failure) to the probability that the event will occur (success).

$$\begin{aligned}\text{Odds against event} &= \frac{P(\text{event fails to occur})}{P(\text{event occurs})} \\ &= \frac{P(\text{failure})}{P(\text{success})}\end{aligned}$$

Six possible outcomes on rolling a die: 1, 2, 3, 4, 5 and 6.

$$P(\text{rolling a } 4) = \frac{1}{6}$$

$$P(\text{failure to roll a } 4) = \frac{5}{6}$$

Determine the odds against rolling a 4 on one roll of a die.

$$\text{Odds against rolling a } 4 = \frac{P(\text{failure to roll a } 4)}{P(\text{rolling a } 4)}$$

$$= \frac{5/6}{1/6}$$

$$= \frac{5}{6} \cdot \frac{6}{1}$$

$$= \frac{5}{1}$$

The odds against rolling a 4 are 5 to 1.

7% of US workers work 35–39 hrs per week.

So $P(\text{person works 35–39 hours per week}) = \frac{7}{100}$

$P(\text{person does not work 35–39 hours per week}) = 1 - \frac{7}{100} = \frac{93}{100}$

The chart shows the percent of U.S. workers who work various hours per week. If one U.S. worker is selected at random, use the chart to determine the odds against the person working 35-39 hours per week.

Up to 34	24%
35 - 39	7%
40 - 48	51%
49 - 59	10%
60 or more	8%

Table: **Hours Worked per Week by U.S. Workers**

odds against the person working 35–39 hours per week $= \frac{P(\text{failure})}{P(\text{success})}$

$= \frac{93/100}{7/100}$

$= \frac{93}{100} \cdot \frac{100}{7} = \frac{93}{7}$

The odds against the person 35–39 hours per week are 93 : 7

Definition:

- The **odds in favor of** an event are expressed as a ratio of the probability that the event will occur to the probability that the event will fail to occur.

$$\begin{aligned} \text{Odds in favor of event} &= \frac{P(\text{event occurs})}{P(\text{event fails to occur})} \\ &= \frac{P(\text{success})}{P(\text{failure})} \end{aligned}$$

A group of parents whose children attended Middlebury College were asked to identify the topic on which their child requested the most advice during the past year. The chart below shows the parents' responses.

Finances	31%
Career planning	20%
Academics	13%
Health and safety	8%
Personal relationships	7%
Campus or community involvement opportunities	1%
Other	11%
None	9%

Table: **Parental Advice**

20% of parents said that their child requested Career planning advice.

P(a child requested career planning advice) $= \frac{20}{100}$

P(a child did not request ~~Career~~ Career planning advice) $= 1 - \frac{20}{100}$
$= \frac{80}{100}$

If one parent from those surveyed is selected at random, use the chart to determine

a) the odds against the parent saying that the child requested career planning advice.

b) the odds in favor of the parent saying that the child requested career planning advice.

a) Odds against a child having requested Career planning advice $= \dfrac{P(\text{failure})}{P(\text{success})}$

$= \dfrac{80/100}{20/100} = \dfrac{4}{1}$ $\boxed{4:1}$

b) The odds in favor of parents saying that their child requested career planning advice are 1 : 4

When odds are given, either in favor of or against a particular event, it is possible to determine the probability that the event occurs and the probability that the event does not occur.

Odds against being admitted

$$= \frac{P(\text{fails to be admitted})}{P(\text{is admitted})}$$

$$= \frac{9}{2}$$

Denominator of both probability of success and probability of failure must be $9 + 2 = 11$

The odds against Robin Murphy being admitted to the college of her choice are 9:2. Determine the probability that

a) Robin is admitted

b) Robin is not admitted.

a) $P(\text{is admitted}) = \frac{2}{9+2} = \frac{2}{11}$

b) $P(\text{is not admitted}) = \frac{9}{9+2}$

$$= \frac{9}{11}$$

a) $P(\text{a } 4) = \frac{1}{6}$

$P(\text{not a } 4) = \frac{5}{6}$

Odds against a 4 $= \frac{5/6}{1/6} = \frac{5}{1}$

b) $P(\text{an odd number}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{not odd}) = \frac{1}{2}$

odds against an odd $= \frac{1/2}{1/2} = \frac{1}{1}$

A die is tossed. Determine the odds against rolling

a) a 4.
b) an odd number
c) a number less than 3
d) a number greater than 4.

c) $P(\text{less than } 3) = \frac{2}{6} = \frac{1}{3}$

$P(\text{a 3 and greater}) = 2/3$

Odds against a number less than 3 $= \frac{2/3}{1/3} = \frac{2}{1}$

d) $P(\text{roll greater than } 4) = \frac{2}{6} = 1/3$

$P(\text{failure to roll greater than } 4) = 2/3$

odds against rolling greater than 4

$= \frac{2/3}{1/3} = \frac{2}{1}$ $\boxed{2:1}$

a) P(pick a queen) $= \frac{4}{52}$

P(failure to pick a queen) $= \frac{48}{52}$

$\boxed{\text{against : } 12:1}$ $\boxed{\text{in favor of } 1:12}$

b) P(pick a heart) $= \frac{13}{52} = \frac{1}{4}$

P(failure to pick a heart) $= \frac{3}{4}$

$\boxed{\text{against } 3:1}$ $\boxed{\text{in favor of } 1:3}$

A card is picked from a standard deck of cards. Determine the odds against and the odds in favor of selecting

a) a queen.

b) a heart.

c) a picture card.

d) a card greater than 5 (ace is low).

c) P(pick a picture) $= \frac{12}{52} = \frac{3}{13}$

P(failure to pick a picture) $= \frac{10}{13}$

$\boxed{\text{against } 10:3}$ $\boxed{\text{in favor of } 3:10}$

d) P(pick a card greater than 5) $= \frac{32}{52} = \frac{8}{13}$

P(failure to pick a card greater than 5) $= \frac{5}{13}$

$\boxed{\text{against } 5:8}$ $\boxed{\text{in favor of } 8:3}$

$$P(\text{red}) = \frac{9}{11}$$

$$P(\text{not red}) = \frac{2}{11}$$

A box contains 9 red and 2 blue marbles. If you select one marble at random from the box, determine the odds against selecting a red marble.

Odds against selecting a red marble $= \dfrac{P(\text{not red})}{P(\text{red})}$

$$= \frac{2/11}{9/11}$$

$$= \frac{2}{9} \quad \boxed{2:9}$$

$$P(\text{win}) = \frac{3}{3+8}$$

$$= \frac{3}{11}$$

The odds in favor of Boris Penzed winning the chicken wing eating contest are 3:8. Determine the probability that Boris will

a) win the contest

b) not win the contest

b) $$P(\text{Boris loses}) = \frac{8}{3+8}$$

$$= \frac{8}{11}$$