12.2 Theoretical Probability

Mathematical Concepts

February 19, 2009

Definitions:

- If each outcome of an experiment has the same chance of occurring as any other outcome, we say that the outcomes are **equally likely outcomes**.
    - The possible outcomes on rolling a die are $\{1, 2, 3, 4, 5, 6\}$. It is equally likely that one would roll any one of the possible numbers.
    - odd and even numbers are another set of equally likely outcomes, that is, $\{1, 3, 5\}$ and $\{2, 4, 6\}$ are the two outcomes.

If an event E has *equally likely outcomes*, the probability of event E may be calculated by

$$P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}}$$

Example 1

a) Six possible equally likely outcomes: 1, 2, 3, 4, 5, 6

The event of rolling a 3 can occur in only one way

$$P(3) = \boxed{\frac{1}{6}}$$

b) The event of rolling an even number can occur in three ways: 2, 4 and 6

$$P(\text{even number}) = \frac{3}{6} = \boxed{\frac{1}{6}}$$

A die is rolled. Find the probability of rolling

a) a 3.
b) an even number.
c) a number greater than 2.
d) a 7.
e) a number less than 7.

c) Numbers greater than 2 : 3, 4, 5, 6 (Four numbers)

$$P(\text{number greater than } 2) = \frac{4}{6} = \boxed{\frac{2}{3}}$$

d) No outcome will result in a 7

$$P(7) = 0$$

e) all outcomes are less than 7

$$P(\text{number less than } 7) = \frac{6}{6} = \boxed{1}$$

Important Facts

1) The probability of an event that cannot occur is 0.
2) The probability of an event that must occur is 1.
3) Every probability is a number between 0 and 1 inclusive; that is, $0 \leq P(E) \leq 1$.
4) The sum of the probabilities of all possible outcomes of an experiment is 1.

In any experiment, an event must either occur or not occur.

The sum of the probability that an event will occur and the probability that it will not occur is 1.

Thus, for any event A

$$P(A) + P(\text{not } A) = 1$$

or we may write

$$P(\text{not } A) = 1 - P(A).$$

Example 3

a) Four 5's

$P(5) = \frac{4}{52} = \boxed{\frac{1}{13}}$

b) $P(\text{not a } 5) = 1 - P(5) = 1 - \frac{1}{13} = \boxed{\frac{12}{13}}$

c) 13 diamonds $\therefore P(\text{diamond}) = \frac{13}{52} = \boxed{\frac{1}{4}}$

Consider a standard deck of 52 playing cards consisting of four suits; hearts, clubs, diamonds and spades. Each suit has 13 cards, including numbered cards ace (1) through 10 and three picture (or face) cards, the jack, the queen, and the king. Hearts and diamonds are red cards; clubs and spades are black cards. There are 12 picture cards, consisting of 4 jacks, 4 queens, and 4 kings. One card is to be selected at random from the deck of cards. Find the probability that the card selected is

a) a 5.
b) not a 5.
c) a diamond
d) a jack or queen or king (a picture card).
e) a heart and a club
f) a card greater than 6 or less than 9.

d) 3 cards of each, giving 12 picture cards

$P(\text{jack or queen or king}) = \frac{12}{52} = \boxed{\frac{3}{13}}$

e) the word 'and' means <u>both</u> events must occur

Not possible

So $P(\text{heart \& club}) = 0$

f) $P(>6 \text{ or } <9) = \frac{8}{52} = \boxed{2/13}$