11.3 Compound Interest

Mathematical Concepts

January 15, 2009

RECALL: *Simple Interest* is calculated once for a period of a loan or investment using the formula

$$i = prt.$$

- The interest paid on savings accounts at most banks is compound interest.
- A bank computes the interest periodically (for example, daily or quarterly) and adds this interest to the original principal.
- The interest for the following period is computed by using the new principal (original principal plus interest).

Interest that is computed on the principal and any accumulated interest is called **compound interest**.

Example 1

First Quarter:

$i = prt$

$= \$1000 \times 0.02 \times 0.25$

$= \$5.00$

$A = \$1000 + \$5 = \$1005$

Second Quarter:

$i = \$1005 \times 0.02 \times 0.25 \approx \$5.03$

$A = \$1005 + \$5.03$

$= \$1010.03$

Marjorie Thrall recently won the $1000 first prize in a raffle contest. Marjorie deposits the $1000 in a 1-year certificate of deposit paying 2.0% compounded quarterly. Find the amount, A, to which the $1000 will grow in 1 year.

3rd Quarter: principal $= \$1010.03$

$i = \$1010.03 \times 0.02 \times 0.25 \approx \$5.05$

$A = \$1010.03 + \$5.05$

$= 1015.08$

4th Quarter:

$i = \$1015.08 \times 0.02 \times 0.25$

$\approx \$5.08$

$A = \$1015.08 + \$5.08$

$= \boxed{\$1020.16}$

COMPOUND INTEREST FORMULA

$$A = p\left(1 + \frac{r}{n}\right)^{nt}$$

where

- A is the amount that accumulates in the account,
- p is the principal,
- r is the annual interest rate as a decimal
- n is the number of compounding periods per year,
- t is the time in years.

Example 2

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P = \$5600$

$r = 7.5\% = 0.075$

$n = 12$

$t = 10$

Kathy Mowers invested $5600 in a savings account with an interest rate of 7.5% compounded monthly. If Kathy makes no other deposits into this account, determine the amount in the account after 10 years.

$$A = 5600\left(1 + \frac{0.075}{12}\right)^{12(10)}$$

$$= 5600(1 + 0.00625)^{120}$$

$$\approx 5600(2.1120646)$$

$$\approx \boxed{\$11,827.56}$$

Example 3

$n = 2, r = 0.08, t = 3$

$$A = p\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 650\left(1 + \frac{0.08}{2}\right)^{(2)(3)}$$

$$= 650(1.04)^6$$

$$\approx \$822.46$$

Calculate the interest on \$650 at 8% compounded semiannually for 3 years, using the compound interest formula.

$$\text{Interest} = \$822.46 - \$650$$

$$= \boxed{\$172.46}$$

The **effective annual yield** or **annual percentage yield (APY)** is the simple interest rate that gives the same amount of interest as a compound rate over the same period of time.

a) $n = 360$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1\left(1 + \frac{0.08}{360}\right)^{(360)(1)}$$

$$\approx 1.0833$$

$$i = A - 1$$

$$\approx 1.0833 - 1 = 0.0833$$

effective annual yield about 8.33%

Determine the annual percentage yield or the effective annual yield for $1 invested for 1 year at

a) ▸ 8% compounded daily

b) ▸ 6% compounded quarterly

$n = 4$, $r = 0.06$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1\left(1 + \frac{0.06}{4}\right)^{(4)(1)}$$

$$\approx 1.0614$$

$$i = A - 1$$

$$\approx 1.0614 - 1 \approx 0.0614$$

effective annual yield is about 6.14%