CSC 300  Test #2  Due:  1:30 pm., 4/29/15

This is a take home test. You may use your textbook, course notes, the library, and a computer in any way that you see fit, as long as appropriate bibliographic references are given. You may not consult any living person other than your instructor.

(5)  1. The Fibonacci sequence is defined by \( F(0) = 0, F(1) = 1, \) and \( F(n) = F(n-1) + F(n-2) \) for \( n \geq 2. \) Show that it satisfies \( F(n+4) = 3 F(n+2) - F(n), \) for all \( n \geq 1. \)

(10)  2. Prove that if 51 positive integers between 2 and 99 (inclusive) are chosen then one of them must divide another one.

(15)  3. Theorem 2, page 410, gives the well-known formula for calculating the Binomial coefficients. This formula is not the best, however, for calculating large binomial coefficients, since the factorial function grows so fast. Use Pascal's identity:

\[
\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}
\]

and recursion or Pascal's triangle to create an accurate way to calculate large binomial coefficients. Use your program to calculate values of \( \binom{n}{\lfloor n/2 \rfloor} \) for as large a value of \( n \) as you can.

(15)  4. The Traveling Salesman Problem is to find the shortest route a salesman should take to visit each city in a set of cities exactly once and return to his starting point. Consider the following table of distances among major Kentucky cities:

<table>
<thead>
<tr>
<th></th>
<th>Ashland</th>
<th>Bowling Green</th>
<th>Covington</th>
<th>Hopkinsville</th>
<th>Lexington</th>
<th>Louisville</th>
<th>Owensboro</th>
<th>Paducah</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ashland</td>
<td>-</td>
<td>278</td>
<td>135</td>
<td>334</td>
<td>123</td>
<td>197</td>
<td>303</td>
<td>385</td>
</tr>
<tr>
<td>Bowling Green</td>
<td>278</td>
<td>-</td>
<td>218</td>
<td>63</td>
<td>155</td>
<td>116</td>
<td>71</td>
<td>153</td>
</tr>
<tr>
<td>Covington</td>
<td>135</td>
<td>218</td>
<td>-</td>
<td>274</td>
<td>82</td>
<td>102</td>
<td>208</td>
<td>325</td>
</tr>
<tr>
<td>Hopkinsville</td>
<td>334</td>
<td>63</td>
<td>274</td>
<td>-</td>
<td>211</td>
<td>172</td>
<td>99</td>
<td>76</td>
</tr>
<tr>
<td>Lexington</td>
<td>123</td>
<td>155</td>
<td>82</td>
<td>211</td>
<td>-</td>
<td>74</td>
<td>180</td>
<td>262</td>
</tr>
<tr>
<td>Louisville</td>
<td>197</td>
<td>116</td>
<td>102</td>
<td>172</td>
<td>74</td>
<td>-</td>
<td>106</td>
<td>223</td>
</tr>
<tr>
<td>Owensboro</td>
<td>303</td>
<td>71</td>
<td>208</td>
<td>99</td>
<td>180</td>
<td>106</td>
<td>-</td>
<td>150</td>
</tr>
<tr>
<td>Paducah</td>
<td>385</td>
<td>153</td>
<td>325</td>
<td>76</td>
<td>262</td>
<td>223</td>
<td>150</td>
<td>-</td>
</tr>
</tbody>
</table>

With 8 cities, there are 7! routes starting at a given city and returning to that city. A brute force solution to this problem is to calculate the total distance of each of these 7! routes and thus determine the minimum. Use your NextPermutaion procedure from Homework Assignment #8 to generate these 7! routes and calculate the shortest route(s) starting and ending in Paducah.
5. Solve the recurrence relation \( a_n = \frac{(a_{n-1})^2}{a_{n-2}} \), if \( a_0 = 1 \) and \( a_1 = 2 \). [Hint: Take the logarithm base 2 of both sides and let \( b_n = \log_2(a_n) \), for \( n = 0, 1, 2, \ldots \)]

6. Let \( f \) be a function with domain \( A \). The relation, \( \sim \), defined by \( x \sim y \) if and only if \( f(x) = f(y) \), is called the kernel relation of \( f \).
   
   (a) Show that \( \sim \) is an equivalence relation on \( A \).
   
   (b) Determine the kernel relation of McCarthy's function defined for positive integers \( n \) by:

\[
M(n) = \begin{cases} 
  n - 10, & \text{if } n \geq 101 \\
  M(M(n + 11)), & \text{if } n \leq 100
\end{cases}
\]

7. Complete Computer Projects 10, 11, 13, and 14, on page 638. Use these routines to determine the smallest equivalence relation that contains the relation \( R = \{(1,3), (3,5), (5,7), (2,1), (4,2), (6,4), (8,9), (10, 10)\} \).

8. In Problem 1 on Test 1, you created a combinatorial circuit that implemented the proposition:

\[(p \lor q \lor \neg r) \land (p \lor \neg q \lor \neg s) \land (p \lor \neg r \lor \neg s) \land (\neg p \lor \neg q \lor \neg s) \land (p \lor q \lor \neg s).\]

   (a) Construct a sum of products expression for the Boolean function represented by this product of sums expression. [Hint: A slight modification of the program from Problem 2 on Test #1 will help you create a truth table]

   (b) Use a Karnaugh map to simplify this expression.

   (c) Draw the circuit based upon this simplified expression.