CSC 300 Test #2 Due: 1:30 pm., 4/29/15

This is a take home test. You may use your textbook, course notes, the library, and a computer in any way that you see fit, as long as appropriate bibliographic references are given. You may <u>not</u> consult any living person other than your instructor.

(5) 1. The Fibonacci sequence is defined by F(0) = 0, F(1) = 1, and F(n) = F(n-1) + F(n-2) for n $\geq$ 2. Show that it satisfies F(n+4) = 3 F(n+2) – F(n), for all n $\geq$ 1.

(10) 2. Prove that if 51 positive integers between 2 and 99 (inclusive) are chosen then one of them must divide another one.

(15) 3. Theorem 2, page 410, gives the well-known formula for calculating the Binomial coefficients. This formula is not the best, however, for calculating large binomial coefficients, since the factorial function grows so fast. Use Pascal's identity:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$ and recursion

or Pascal's triangle to create an accurate way to calculate large binomial coefficients. Use your program to calculate values of $\binom{n}{\lfloor \frac{n}{2} \rfloor}$ for as large a value of n as you can.

(15) 4. The Traveling Salesman Problem is to find the shortest route a salesman should take to visit each city in a set of cities exactly once and return to his starting point. Consider the following table of distances among major Kentucky cities:

	Ashland	Bowling Green	Covington	Hopkinsville	Lexington	Louisville	Owensboro	Paducah
Ashland	-	278	135	334	123	197	303	385
Bowling Green	278	-	218	63	155	116	71	153
Covington	135	218	-	274	82	102	208	325
Hopkinsville	334	63	274	-	211	172	99	76
Lexington	123	155	82	211	-	74	180	262
Louisville	197	116	102	172	74	-	106	223
Owensboro	303	71	208	99	180	106	-	150
Paducah	385	153	325	76	262	223	150	-

With 8 cities, there are 7! routes starting at a given city and returning to that city. A brute force solution to this problem is to calculate the total distance of each of these 7! routes and thus determine the minimum. Use your NextPermutaion procedure from Homework Assignment #8 to generate these 7! routes and calculate the shortest route(s) starting and ending in Paducah.

(10) 5. Solve the recurrence relation $a_n = (a_{n-1})^2 / a_{n-2}$, if $a_0 = 1$ and $a_1 = 2$. [Hint: Take the logarithm base 2 of both sides and let $b_n = \log_2(a_n)$, for n = 0, 1, 2, ...]

(15) 6. Let f be a function with domain A. The relation, ~, defined by x ~ y if and only if f(x) = f(y), is called the kernel relation of f.

(a) Show that ~ is an equivalence relation on A.

(b) Determine the kernel relation of McCarthy's function defined for positive integers n by:

$$M(n)=\begin{cases} n-10, & \text{if } n \geqslant 101 \\ M(M(n+11)), & \text{if } n \leq 100 \end{cases}$$

(15) 7. Complete Computer Projects 10, 11, 13, and 14, on page 638. Use these routines to determine the smallest equivalence relation that contains the relation R = {(1,3), (3,5), (5,7), (2,1), (4,2), (6,4), (8,9), (10, 10)}.

(15) 8. In Problem 1 on Test 1, you created a combinatorial circuit that implemented the proposition:

$(p \lor q \lor \neg r) \land (p \lor \neg q \lor \neg s) \land (p \lor \neg r \lor \neg s) \land (\neg p \lor \neg q \lor \neg s) \land (p \lor q \lor \neg s)$.

(a) Construct a sum of products expression for the Boolean function represented by this product of sums expression. [Hint: A slight modification of the program from Problem 2 on Test #1 will help you create a truth table]

(b) Use a Karnaugh map to simplify this expression.

(c) Draw the circuit based upon this simplified expression.