

This is a take home test. You may use textbooks, course notes, the library, and a computer in any way that you see fit, as long as appropriate bibliographic references are given. You may not consult any living person other than your instructor.

- (10) 1. Draw a combinatorial circuit that implements the proposition:
 $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
- (15) 2. Write a program that, given a compound proposition, determines whether it is satisfiable by checking its truth value for all possible assignments of truth values to its propositional variables. Use your program to solve Exercise 62, part c on page 36 of our textbook.
- (10) 3. Solve the recurrence relation $a_n = 3 a_{n-1} + 1$, $a_0 = 1$.
- (10) 4. Let A and B be subsets of the finite universal set U . Show that
 $|A \cap B| = |U| - |A| - |B| + |A \cup B|$.
- (15) 5. Write a program that, given an $m \times k$ Boolean Matrix A and a $k \times n$ Boolean Matrix B , calculates and prints the Boolean product of A and B . Verify your program by applying it to Exercise 28 on page 185
- (10) 6. Devise an algorithm that finds the closest pair of integers in a sequence of n integers, and determine the worst-case complexity of your algorithm. [Hint: Sort the sequence. Use the fact that sorting can be done with worst-case time complexity $O(n \log n)$.]
- (15) 7. Write a program that, given an ordered list of integers, determines the position of an integer in the list using a binary search. Test your program by filling an array of size 100,000 with random integers between 1 and 200,000, sorting it and then searching for 10 values chosen at random from 1 to 200,000. Your search routine should return -1 if the search value is not found.
- (15) 8. Write a program that, given integers n and b , each greater than 1, finds the base b expansion of n . See Algorithm 1 on page 249. Your program should use the digits 0 through 9 and the letters A through Z for bases of 36 or less. Use your program to express the integer 42 using each base from 2 to 36.