Math 505–01: Abstract Algebra I

Spring 2008

The twenty-three year old Felix Klein in his famous Erlanger Programm [1872] proposed to classify geometries by their automorphisms. He hit on something fundamental here: in a sense, structure is whatever is preserved by automorphisms.

— *W. Hodges,* Model Theory*, 1993*

Instructor: Wesley Calvert
Office: Faculty Hall 6C-2
Office Hours: Official (guaranteed) hours Tu 8:30-10:00, Th 1:30-3:30, Fr 8:30-10:00, and 1:30-4:30; also make an appointment or come see me
Office Phone: 809-2503
Home Phone: 761-3751
e-mail: `wesley.calvert@murraystate.edu`
Web Page: `http://campus.murraystate.edu/academic/faculty/wesley.calvert/teaching/505s08/`

Course Goals

The central goal of this course is to allow students considerable experience in thinking like a mathematician. There are two parts of this that deserve special emphasis.

The first is that mathematicians have a particular way of investigating questions. What we do is generate some examples and try to detect a pattern (i.e. a *conjecture*). When we have a guess at a pattern, then we try to explain why that pattern should always hold (i.e. *prove* the conjecture). Proofs without examples are usually impossible and often worthless. Examples without proofs are often misleading. The happy marriage of the two is the mathematical method, which we will use extensively.

The second aspect of mathematical thinking on which we will concentrate is, as the epigraph indicated, the identification of structure, and its relationship to the homomorphisms into and out of a structure, especially the automorphisms. You can tell quite a lot about a number by the effect of adding its multiplicative inverse to the integers. You can also tell a lot from the extent to which it is determined by the polynomials it satisfies.

A subsidiary goal is to allow the student to become familiar with the language and basic properties of algebraic structures, especially rings and fields. These objects are indispensable in modern mathematics, physics, and computer science, and are inseparable from their roots in the central problem of all algebra, from Algebra I through front-line research: Solving polynomials.

Course Content

We will begin roughly where MAT 421 left off, with the study of rings and fields. The "blocks" by which rings may be divided are called "ideals," and we will spend quite a lot of time on them. This will have important consequenses for factorization of both numbers and polynomials.

Later in the course, we will approach vector spaces from a rather lofty viewpoint, and use them, in connection with what we know of rings and fields, to address some of the great problems of ancient mathematics:

1. Given a straight-edge and compass and an angle, construct another angle of one-third the size.
2. Given a straight-edge and compass and a circle, construct a square with the same area.
3. Give a formula — using only the four basic operations plus the square root — which will give the solution to an arbitrary degree five polynomial (analogous to what the quadratic formula does for degree two).

All three of these are proven impossible by the following method:

1. Look at the set of all numbers of the form $a_0 + a_1 x + a_2 x^2 + \cdots$, where all of the a_i's are rational and each x_i is either the length of something in the construction or a root of the equation.
2. Find the structure of the group of automorphisms of this ring.
3. Show that such a group is incompatible with what was to be done.

We will spend some time talking about these problems, although we may not see the solutions to all of them in full detail.

Course Activities

Homework will be assigned daily or almost daily and will be collected weekly, on Fridays (unless otherwise announced). There will be a truckload of it, and that's not because I'm sadistic. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second, some individual problems will have to be thought about, played with, put down, and revisited. You *must* allow time for this. Third (and most importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

The class will meet on each Monday, Wednesday, and Friday at 10:30am. A typical meeting will begin a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this.

When anyone is speaking, think: Do I understand what is being said? Do I agree that it is correct? Can I think of a different way to explain it? Can I answer the question being asked (or the one that is being answered)? You should be careful to ask any questions you have (a student often erroneously thinks that he or she is the only one who wonders; a teacher often wonders why the only person bright enough to know they have a question would be shy about it), and to take full part in answering the questions of others, when you are able. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Joseph A. Gallian, *Contemporary Abstract Algebra*, 6ed, Houghton Mifflin, 2006. Bookstore Price (Used) $90.50.

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. Gallian is widely and justly known in mathematical circles as an effective communicator in oral and written English. This will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher. It will also be the source of many of the homework problems. Be careful of this, though: One can easily get the impression from the book that the right way to think about things is to memorize some formula or some procedure. In practice, if you try to do this with everything we will learn in the approximately forty-five hours we have together in class this semester, plus the time spent outside of class, you will likely be overwhelmed and miserable. Better is to try and find the big ideas, and re-build everything else as you need it. You'll do better with this class and with later ones, and you'll not have to memorize nearly as much (i.e. it's easier). Gallian recommends memorizing definitions and theorems. I will recommend this on a few particularly important definitions and theorems, but certainly not all of them.

Each student will also complete an individual research project, on a topic mutually agreeable to the student and the instructor. Topics must be approved by February 18, and each student must turn in an outline of their findings

by April 1. Each student must schedule an apointment with the instructor on or before April 18 to discuss final preparations. Each student will give an oral presentation to the class, and graduate students will submit a written report. Graduate students must also make approved (significant) use of at least one approved (technical) resource beyond the textbook. Graduate students must submit a draft of the written report on or before April 18, and the final report on or before the day of the final exam. More information on the projects will follow.

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 8, March 14, and April 18. In addition, there will be a final exam at 10:30am on Thursday, May 8, in our usual room. The final will test your ability to do all of the things we have worked on in class, with particular, but not exclusive, emphasis on what we have done since the last test. Of course, no cooperation can be permitted on exams.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

Good writing is not a central goal of this class, but it is necessary for achieving the central goals of this class. All homework and test solutions must be well-written. A writing guide for the course will be made available under separate cover, and will be posted on the course web page. This is not only friendly advice about how to learn the material and have a good life and a good career (although I do also offer it in that spirit), but is also a requirement of all assignments in this course.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem does absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

Grading

Grades will be calculated from the following sources:

Undergraduate		**Graduate**	
Homework	200pts	Homework	200pts
Research Project	100pts	Research Project (oral)	100pts
		Research Project (written)	100pts
Regular Exams (3 @ 100pts/ea.)	300pts	Regular Exams (3 @ 100pts/ea.)	300pts
Final Exam	100pts	Final Exam	100pts
	600		700

I regret that I will not be able to provide a detailed reading of every problem I ask you to submit. The truth is, to learn the material, you need to do more homework than I could possibly read. On each assignment, I will grade a small but representative sample of the problems. If you would like more detailed feedback on another problem, I would be glad to give it.

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, I recently noticed that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. Moreover, since you can use the book, talk with friends, talk with a tutor, ask me how to do the problem, etc., *everyone should receive a grade of near 100% on the homework*. It is depressing how rarely this happens. Indeed, due largely to negligence in completing and turning in all of the assigned problems, many students find that their homework grade instead brings their grade in the course down. Don't let this happen to you.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

Percent of total	Grade
90–100	A
80–89	B
70–79	C
60–69	D
≤ 59	E

Statement of Affirmative Action and Equal Opportunity

Murray State University does not discriminate on the basis of race, color, national origin, sex, religion, marital status, age, or disability in employment, admission, or the provision of services, educational programs and activities, and provides, upon request, reasonable accommodation including auxiliary aids and services necessary to afford individuals with disabilities an equal opportunity to participate in all programs and activities. For information regarding nondiscrimination policies contact the Office of Equal Opportunity, 270-809-3155.

I personally and professionally take this matter very seriously. If I can be of any help to you in this or any other area, please let me know. Moreover, it is my experience that often students don't know what kind of help is available to them, particularly in the area of accommodations for disabilities. I would be honored to help you find out.

Academic Honesty

If, which may it never happen, academic dishonesty occurs in this course, in the determination of the instructor, grade penalties may be imposed. Such a penalty shall not be less than a grade of zero points on the assignment on which the dishonesty takes place, and in serious or repeated offenses may be failure of the class. See also the Policy on Academic Honesty in the University Bulletin.

Administrative Notes

This course carries three hours of credit.

MAT 421 is prerequisite for this class. Any student not meeting this requirement is *strongly* advised to delay taking this class until it is satisfied. This is one of the hardest courses in the mathematics curriculum. A ten kilometer road race is hard enough if you've been training for it. Don't try it if you haven't run a full mile.