# Math 150–02: Algebra and Trigonometry

Spring 2008

*A possible explanation of the physicist's use of mathematics to formulate his [*sic*] laws of nature is that he is a somewhat irresponsible person. As a result, when he finds a connection between two quantities which resembles a connection well-known from mathematics, he will jump at the conclusion that the connection is that discussed in mathematics simply because he does not know of any other similar connection. It is not the intention of the present discussion to refute the charge that the physicist is a somewhat irresponsible person. Perhaps he is. However, it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language.*

— *Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," 1960*


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## Course Goals

The goal of this course is to provide a background in algebra and trigonometry which will serve as the foundation for study of mathematics and the sciences. If you've read a newspaper in the past few years, you've probably noticed that the world is getting more technical. Mathematical modeling is now used not only to design buildings and computers, but to design medicines and financial securities, and to analyze terrorist networks, to name but a few. The algebra that once allowed for the solution of abstract equations now underlies every computer security system in the world. If you want to have a part in the scientific, technical, business, or political life of the next fifty years, you must be mathematically literate.

Not to mention, of course, that mathematics is one of the most beautiful bodies of knowledge that human reasoning has ever produced. This course is an entryway.

## Course Content

The University's course catalogue describes this course as "An intensive study of college algebra and trigonometry. A combination of MAT 140 and MAT 145, it is a faster-paced course for students with some familiarity with the subjects. MAT 150 may be used as a 'refresher course' to prepare students for MAT 250." In the rest of this section, I will attempt to say just what "college algebra and trigonometry" we will study.

The content of the course will center around functions. A function, quite simply, is a way to express how one thing depends on another. Examples include the relationship between depth of a car windshield and the distance from the centerline of the car, that between the number of users on a computer network and the probability of a serious security breach, and that between a first-grader's performance on standardized tests and the same student's performance in high school class assignments.

We will begin by studying algebraic equations and the geometry that corresponds to each one. We will see how to translate from one domain (mathematicians and philosophers of math sometimes say "category") to the other, and how to use information that is easily available on one side to answer questions on the other. We will then explore how to express relationships as functions, which will enable us to have very exact ways to state intuitively obvious properties like symmetry.

At that point, we will turn to the special properties of some particularly important sorts of functions. Polynomials (and their close cousins, rational functions) are regarded by many as the "nicest" of all functions. If you can describe some phenomenon by a polynomial function (as Newton did for classical mechanics), then it becomes as well understood as anything in our world. It is for that reason that machines (robotic arms, car windshield wipers, etc.)

are often designed to have their motion governed by polynomials. Time permitting, we will take a distant glance at this point at one of the biggest open problems in mathematics today, which lies in this territory.

At this point, we will move into the territory of the "transcendental" functions. Among these, the exponential and logarithmic functions are the closest to the ideal of polynomials. They tend to be very good models for systems whose behavior in the next unit of time depends primarily on its current state. For instance, the number of people who will be afflicted tomorrow with the cold that's spreading around campus depends primarily on how many people have a cold today. If (which may it never happen to you) you allow a balance to remain on your credit card without paying it off, so that it accrues interest, the amount you owe next month will depend in an important way on how much you owe this month. And what was the deal with that McDonalds coffee lawsuit, anyway?

It often happens that one wishes to describe periodic behavior. Body temperature of a healthy person, wave motion, and economic indicators are typical examples. Also, it is useful to describe the relationship between angular and linear measurements (Where should you sit in a movie theater to maximize the apparent size of the screen?). Trigonometric functions accomplish both goals.

Unfortunately, the interaction of trigonometric functions with normal algebraic operations is rather involved. We will spend a good deal of time dealing with these interactions. On the other hand, the trigonometric functions interact with one another in rather beautiful and often simple ways (many involve nothing more than the Pythagorean theorem). One of the advantages to venturing into this study is a chance to do something that is central to active mathematics, but usually absent in high school (and often college) courses: we will prove things.

Some particular topics in the course are central. Others are less central, and if time constraints demand it, we may omit some rather than doing everything badly.

## Course Activities

Homework will be assigned daily or almost daily and will be collected weekly, on Fridays (unless otherwise announced). There will be a truckload of it, and that's not because I'm sadistic. The most common thing in all of mathematics — I do it myself, as does every other mathematician I know — is to see somebody else doing a problem and say, "Yes, yes, of course. I understand completely," and then walk away and realize that we had no idea at all what was going on. Homework is your guard against this. If you really understand how to do the homework, you're generally in pretty good shape. If you can't, you've got plenty of time to figure it out, ask me, ask a friend, or take whatever other action you see fit.

Homework will always be due at 4:30 on the appointed day. You are, of course, welcome to turn it in when you come to class. If you wish, though, you may continue to work on it, and may deliver it to my office or my department mailbox.

Cooperation on homework is strongly encouraged. There will almost certainly be problems on which it is necessary. Talk with each other, talk with me, talk with friends, use any resource. It is important, however, to be sure that you understand the solution you present. In designing the tests, I will assume thorough familiarity with all homework problems due before the date of the exam.

You are also encouraged to visit me in my office (see note on office hours above) or to call or e-mail me. To be more clear: It's a hard class. I'd like to see you do well in it. I'd love to talk with you and to help you in any way that I can.

It is wise to work on the homework as it is assigned, for a couple of reasons. First, there will be enough of it that it will not be practical to just sit down and do the whole week's worth in an evening. Second (and more importantly), the material builds on itself, so that a few days without working through at least some of the problems may find you feeling a little lost.

The class will meet on each business day at 12:30pm. A typical meeting will begin a discussion of any questions folks have, with procedural matters treated first. This will be followed by a discussion of new material (often in the form of problems, on which students will work in groups) and typically an assignment of new homework.

You should be in every class meeting, and should make sure that you are actively engaged. It goes without saying that when a problem is assigned for group work, you must do it. If you wait for me to tell you how to do it, then by the time I talk about the solution with the class, everybody else will understand it and will be ready to ask about issues you haven't encountered, and you will be lost. Don't do this.

When anyone is speaking, think: Do I understand what is being said? Do I agree that it is correct? Can I think of a different way to explain it? Can I answer the question being asked (or the one that is being answered)? You should be careful to ask any questions you have (a student often erroneously thinks that he or she is the only one who wonders; a teacher often wonders why the only person bright enough to know they have a question would be shy about it), and to take full part in answering the questions of others, when you are able. You should also feel free to be wrong. We all will be at some point in the class. That's why we gather together, instead of just reading the book

on our own: we can help one another understand better, and we can try out ideas on each other, even if we aren't quite sure of them.

Text: Sullivan and Sullivan, *Algebra & Trigonometry, Enhanced with Graphing Utilities*, Fourth Edition, Prentice Hall, 2006. List price, $128; Bookstore Price $120.

Be warned. The bookstores have been known to offer some other books as "recommended" for math courses. They are recommended by the bookstore, not by the math department, and not by me. I don't particularly recommend against them (since I have little idea what they'll be), but let the buyer be ware.

The text makes a great effort — and a successful one at most points — to be readable. It will provide an important opportunity to get an explanation in a different voice (at times very different) than that of your beloved teacher. It will also be the source of the bulk of the homework problems. Be careful of this, though: One can easily get the impression from the book that the right way to think about things is to memorize some formula or some procedure. In practice, if you try to do this with everything we will learn in the approximately seventy-five hours we have together in class this semester, plus the time spent outside of class, you will likely be overwhelmed and miserable. Better is to try and find the big ideas, and re-build everything else as you need it. You'll do better with this class and with later ones, and you'll not have to memorize nearly as much (i.e. it's easier).

There will also be some exams. Each exam will be preceded by a review sheet indicating *exactly* what material will be covered, an in-class review session, and an out-of-class review session. Exams will be given in the regularly scheduled class time and place on February 1, February 22, March 28, and April 17. In addition, there will be a final exam at 10:30am on Wednesday, May 7, in our usual room. The final will test your ability to do all of the things we have worked on in class, with particular, but not exclusive, emphasis on what we have done since the last test. Of course, no cooperation can be permitted on exams.

The general philosophy is that class sessions and homework will be very hard and tests will be pretty easy (assuming, of course, that you've suffered through the class meetings and homework leading up to them). Again, my goal with the homework is to help you to understand the material so well that you're unhappy with me for giving such a boring (easy) test.

In all activities for this class, make sure that you *do something*. It is depressing how often students who probably know something relevant to a problem does absolutely nothing, allowing no opportunity to receive credit on the part they actually know.

## On Calculators

You will need a graphing calculator for this course. I will have a TI-83+, produced by Texas Instruments, and am also familiar with the operation of their TI-85 model, having used one for years. On other models I will make a good faith effort to assist you, but cannot be finally responsible for teaching you their use.

## Grading

Grades will be calculated from the following sources:

| | |
|---|---|
| Homework | 100pts |
| Regular Exams (4 @ 100pts/ea.) | 400pts |
| Final Exam | 100pts |
| | 600 |

I regret that I will not be able to provide a detailed reading of every problem I ask you to submit. The truth is, to learn the material, you need to do more homework than I could possibly read. On each assignment, I will grade a small but representative sample of the problems. If you would like more detailed feedback on another problem, I would be glad to give it.

Failure to attend class regularly will certainly adversely affect your grades on each of these factors. For instance, while I do not artificially lower grades for bad attendance, I recently noticed that almost all grades below C- that have been achieved in classes that I have taught have been associated with significant attendance problems.

In like manner, you should not underestimate the impact of your homework. Not only does the experience of the homework problems impact your test grades, but the homework itself is a considerable portion of the grade in the class. Moreover, since you can use the book, talk with friends, talk with a tutor, ask me how to do the problem, etc., *everyone should receive a grade of near 100% on the homework*. It is depressing how rarely this happens. Indeed,

due largely to negligence in completing and turning in all of the assigned problems, many students find that their homework grade instead brings their grade in the course down. Don't let this happen to you.

In all work done for this class, work is more important than answers. A correct answer without correct work (or worse, with work that does not match the answer) is not worth much at all, while generally correct work with an incorrect answer is almost as good as being completely right. Thus, getting the right answer does not guarantee a good grade on the problem, and getting a wrong answer does not guarantee a bad one.

I will make the following guarantees about letter grades. I may decide to lower these criteria (i.e. give a higher grade than the one shown here, if I see that the questions were hard enough that lower numbers more accurately reflect my true standards), but will never raise them.

| Percent of total | Grade |
|---|---|
| 90–100 | A |
| 80–89 | B |
| 70–79 | C |
| 60–69 | D |
| $\leq 59$ | E |

## Statement of Affirmative Action and Equal Opportunity

Murray State University does not discriminate on the basis of race, color, national origin, sex, religion, marital status, age, or disability in employment, admission, or the provision of services, educational programs and activities, and provides, upon request, reasonable accommodation including auxiliary aids and services necessary to afford individuals with disabilities an equal opportunity to participate in all programs and activities. For information regarding nondiscrimination policies contact the Office of Equal Opportunity, 270-809-3155.

I personally and professionally take this matter very seriously. If I can be of any help to you in this or any other area, please let me know. Moreover, it is my experience that often students don't know what kind of help is available to them, particularly in the area of accommodations for disabilities. I would be honored to help you find out.

## Academic Honesty

If, which may it never happen, academic dishonesty occurs in this course, in the determination of the instructor, grade penalties may be imposed. Such a penalty shall not be less than a grade of zero points on the assignment on which the dishonesty takes place, and in serious or repeated offenses may be failure of the class. See also the Policy on Academic Honesty in the University Bulletin.

## Administrative Notes

This course carries five hours of credit.

An ACT math standard score of at least 22 is prerequisite for this class. Any student not meeting these requirements is *strongly* advised to delay taking this class until they are satisfied. This is two courses compressed into one. It is intensive. It is fast. It is hard. A ten kilometer road race is hard enough if you've been training for it. Don't try it if you haven't run a full mile.

A student who receives credit for MAT 150 may not receive credit for MAT 130, 140, or 145. Credit for the combination of MAT 140 and MAT 145 will substitute for MAT 150. Consequently, if you have taken MAT 150 before and have received an unsatisfactory grade, I strongly recommend that you take MAT 140 and MAT 145. This sequence will help you to learn the material better, will be easier, and will still replace the previous grade from MAT 150.