Errata and comments on a Murray State University master's thesis

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This document contains errata and other remarks on the Murray State University master's thesis

Picturing Representations of Simple Lie Algebras of Rank Two, by Marti L. McClard, May 2000.

This thesis was written under the direction of Robert G. Donnelly.

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p. 1 Comment: McClard's thesis was the beginning of an extensive poset-theoretic study of the irreducible representations of the rank two semisimple Lie algebras. The subsequent work, some of which is still in progress, appears in [DLP1], [DLP2], [Alv], [ADLP], [ADLMPPW], [Don2].

The main goal of McClard's thesis was to demonstrate that certain distributive lattices orderings of Littelmann's G_2-tableaux (see [Lit]) are "suitable" for seeking explicit presentations of the irreducible representations of the simple Lie algebra G_2. These distributive lattice orderings were proposed by Donnelly. The lattices are denoted $L_G^{Lit}(2,\lambda)$ in McClard's thesis, where $\lambda = (a,b)$ is always a pair of nonnegative integers corresponding to a dominant weight $a\omega_1 + b\omega_2$. (Here, ω_1 (resp. ω_2) is the highest weight for the fundamental representation of G_2 of dimension 7 (resp. 14).)

Certain necessary (but not sufficient) conditions that make a poset "suitable" for explicitly presenting semisimple Lie algebra actions were laid out in §3.2 of [Don1]. In the recent paper [ADLMPPW], such a poset is called a "splitting poset." A splitting posets is, in a certain sense, a combinatorial model for a Weyl character. A splitting poset that can be used to present semisimple Lie algebra actions in a certain way is called a "supporting graph" in [Don1]. In this language, the main result of McClard's thesis is that for any λ, the distributive lattice $L_G^{Lit}(2,\lambda)$ is a splitting poset for the irreducible G_2-Weyl character corresponding to λ. This result is a consequence of the statement and proof of Theorem 6.2.8 of McClard's thesis. (See also §6.3 of McClard's thesis.) It was further conjectured in the thesis (Conjecture 6.2.3) that $L_G^{Lit}(2,\lambda)$ is a supporting graph for the corresponding irreducible representation of G_2.

It was shown in [DLP1] that indeed each G_2-lattice $L_G^{Lit}(2,(a,0))$ is a supporting graph for the irreducible representation of G_2 with highest weight $a\omega_1$. However, more recently it was shown in [ADLP] that a G_2-lattice $L_G^{Lit}(2,\lambda)$ is a supporting graph for a representation of G_2 if and only if $\lambda = (a,0)$ or $\lambda = (0,1)$. That is, Conjecture 6.2.3 of McClard's thesis is only true for these particular λ.

p. 33, l. -7 Comment: It is Lemma 2.2.3 that is being applied to prove Theorem 5.1.1. So the last line of the proof of Theorem 5.1.1 should read, "By Lemma 2.2.3, $L_A^{GT-left}(n,\lambda)$ is a distributive lattice."

p. 37, l. -2 Erratum: The formula for the number of vertices in P is missing the factor $(a+2b+3)$.

p. 41, l. 2 Typo: "$\ldots wt_{Lit}(\mathbf{s}) + \alpha_i = wt_{Lit}(\mathbf{t}) \ldots$" (The argument "**s**" is missing in the thesis text.)

p. 42, l. 7 Comment: Although it is apparent at this point in the thesis that $L_G^{Lit}(2,\ydiagram{1})$ and $L_G^{Lit}(2,\ydiagram{1,1})$ are distributive lattices, this should be part of the Lemma 6.2.1 statement:

Lemma 6.2.1 *$L_G^{Lit}(2,\ydiagram{1})$, $L_G^{Lit}(2,\ydiagram{1,1})$, $L_G^{Lit}(2,\ydiagram{2,1})$, $L_G^{Lit}(2,\ydiagram{2,2})$, and $L_G^{Lit}(2,\ydiagram{2})$ are distributive lattices. (Here, the order is reverse component-wise comparison.)*

p. 43, l. 8 Comment: In the preparation of [ADLP], this theorem attributed to Littelmann (Theorem 6.2.4) was confirmed by a straightforward counting argument.

p. 47, l. 2 Typo: Should be "Theorem 6.2.2," not "Corollary 6.2.2."

$\boxed{\text{p. 49, l. 1}}$ Erratum: The top line of the second column of the table is missing a factor of 2, as it should be "$2\rho_2(t_j) - l_2(t_j)$."

$\boxed{\text{p. 50, l. 4}}$ Erratum: The top line of the second column of the table is missing a factor of 2, as it should be "$2\rho_1(t_j) - l_1(t_j)$."

$\boxed{\text{p. 52, l. 3}}$ Typo: It should be "$sp(2n, \mathbb{C})$" here, not "$so(2n+1, \mathbb{C})$."

$\boxed{\text{p. 53, l. 1}}$ Typo: It should be "$so(2n+1, \mathbb{C})$" here, not "$sp(2n, \mathbb{C})$."

$\boxed{\text{p. 61}}$ Comment: In Figure B.7, the 2nd vertex from the left on the middle level actually represents *two* vertices. (Some three-dimensionality was lost in typesetting the lattice figure in the LaTeX picture environment.)

$\boxed{\text{p. 62, l. 1}}$ Update: The reference given there became [Don1] below.

$\boxed{\text{p. 62, l. 11}}$ Update: This paper ("Weight bases of Gelfand-Tsetlin type for representations of classical Lie algebras") by A. Molev has appeared in *J. Phys. A: Math. Gen.* **33** (2000), 4143–4168.

References

[Alv] L. W. Alverson II, "Distributive lattices and representations of the rank two simple Lie algebras," Master's thesis, Murray State University, 2003.

[ADLMPPW] L. W. Alverson II, R. G. Donnelly, S. J. Lewis, M. McClard, R. Pervine, R. A. Proctor, and N. J. Wildberger, "Distributive lattices defined for representations of rank two semisimple Lie algebras," to appear in *SIAM J. Discrete Math.*, `arXiv:0707.2421`.

[ADLP] L. W. Alverson II, R. G. Donnelly, S. J. Lewis, and R. Pervine, "Constructions of representations of rank two semisimple Lie algebras with distributive lattices," *Electronic J. Combin.* **13** (2006), #R109 (44 pp).

[Don1] R. G. Donnelly, "Extremal properties of bases for representations of semisimple Lie algebras," *J. Algebraic Combin.* **17** (2003), 255–282.

[Don2] R. G. Donnelly, "Dynkin diagram classification results for distributive lattices satisfying certain structural properties," in preparation.

[DLP1] R. G. Donnelly, S. J. Lewis, and R. Pervine, "Constructions of representations of $\mathfrak{o}(2n+1, \mathbb{C})$ that imply Molev and Reiner-Stanton lattices are strongly Sperner," *Discrete Math.* **263** (2003), 61–79.

[DLP2] R. G. Donnelly, S. J. Lewis, and R. Pervine, "Solitary and edge-minimal bases for representations of the simple Lie algebra G_2," *Discrete Math.* **306** (2006), 1285–1300.

[Lit] P. Littelmann, "A generalization of the Littlewood-Richardson rule," *J. Algebra* **130** (1990), 328–368.