31.3 Integrable Combinations

Most differential equations will not be able to be solved by the Separation of Variables discussed in Section 31.2. Other methods will need to be used.

This section we talk about a method that integrates together certain combinations of basic differentials.

$$d(xy) = x\,dy + y\,dx \qquad \text{(Product Rule)}$$

$$d(x^2 + y^2) = 2x\,dx + 2y\,dy \qquad \text{(Power Rule)}$$

$$\left.\begin{aligned} d\left(\frac{x}{y}\right) &= \frac{y\,dx - x\,dy}{y^2} \\ d\left(\frac{y}{x}\right) &= \frac{x\,dy - y\,dx}{x^2} \end{aligned}\right\} \qquad \text{(Quotient Rule)}$$

p.947

④ $(2y+t)\,dy + y\,dt = 0$

$$2y\,dy + \underbrace{t\,dy + y\,dt}_{} = 0$$

$$2y\,dy + d(ty) = 0$$

$$\int 2y\,dy + \int d(ty) = \int 0$$

$$\boxed{y^2 + ty = C}$$

⑥ $\dfrac{x\,dy - y\,dx}{y^2} + \dfrac{y^2\,dx}{y^2} = \dfrac{0}{y^2}$

$$-\left(\frac{x\,dy - y\,dx}{y^2}\right) + (dx) = 0$$

$$\frac{y\,dx - x\,dy}{y^2} - dx = 0$$

$$d\left(\frac{x}{y}\right) - dx = 0$$

$$\int d\left(\frac{x}{y}\right) - \int dx = 0$$

$$\boxed{\frac{x}{y} - x = C}$$

(8) $(y + \sec(xy))\, dx + x\,dy = 0$

$y\,dx + \sec(xy)\,dx + x\,dy = 0$

$$\frac{\sec(xy)\,dx}{\sec(xy)} + \frac{x\,dy + y\,dx}{\sec(xy)} = \frac{0}{\sec(xy)}$$

$x\,dy + y\,dx = d(xy)$

$$dx + \underbrace{\cos(xy)}_{\cos u}\, \underbrace{d(xy)}_{du} = 0$$

$$\int dx + \int \cos(xy)\, d(xy) = 0$$

$$\boxed{x + \sin(xy) = C}$$

(16) $e^{x+y}(dx + dy) + 4x\,dx = 0$

$u = x + y \qquad du = dx + dy$

$$\int e^{x+y}(dx + dy) + \int 4x\,dx = 0$$

$$\boxed{e^{x+y} + 2x^2 = C}$$

(20) $$\frac{t\,dt + s\,ds}{t^2+s^2} = \frac{2(\cancel{t^2+s^2})\,dt}{\cancel{t^2+s^2}} \qquad t = 1 \text{ when } s = 0$$

$$\frac{2(t\,dt + s\,ds)}{t^2+s^2} = 2(2\,dt)$$

$u = t^2 + s^2$

$du = 2t\,dt + 2s\,ds$

$$\int \frac{du}{u} = \int 4\,dt$$

$\ln(t^2+s^2) = 4t + C$ ← general solution

Find particular solution using $t = 1$ & $s = 0$.

$\ln(1^2 + 0^2) = 4(1) + C \Rightarrow 0 = 4 + C \Rightarrow C = -4$

particular solution is $\boxed{\ln(t^2+s^2) = 4t - 4}$