3.1 Solutions of Differential Equations

Differential Equation is an equation that involves derivatives of an unknown function of one or more variables.

$$y'' = 3 \qquad \frac{d^2y}{dx^2} = 3 \qquad y'' + 2yy' + y = -2$$

The **order** of a differential Equation is the order of the highest derivative in the equation.

$y'' = 3$ order is 2 $\qquad y'' - yy''' + 3 = y$ order is 3

$\frac{d^2y}{dx^2} - 2\frac{d^4y}{dx^4} = 5$ order is 4 $\qquad y'' - 2y'''' = 5$

$y^{(II)} - 2y^{(IV)} = 5$

The **solution** of the differential equation is the function that satisfies the equation.

General Solution - solution containing a number of independent arbitrary constants.

Particular Solution - solution with specific values given to the constants.

p.941 ③ $\frac{dy}{dx} + 2xy = 0$ (1st order) $\qquad y = e^{-x^2} \Rightarrow y' = \frac{dy}{dx} = e^{-x^2}(-2x)$

$= -2xe^{-x^2}$

$-2xe^{-x^2} + 2x\left(e^{-x^2}\right) \stackrel{?}{=} 0$ ✓

$y = e^{-x^2}$ is a particular solution

(5) $y'' + 3y' - 4y = 3e^{x}$

2nd order

$y = c_1 e^{x} + c_2 e^{-4x} + \frac{3}{5} x e^{x}$ Prod Rule

$y' = c_1 e^{x} - 4c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{3}{5} e^{x}$

$y'' = c_1 e^{x} + 16 c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{3}{5} e^{x} + \frac{3}{5} e^{x}$

$= c_1 e^{x} + 16 c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{6}{5} e^{x}$

$y'' + 3y' - 4y = 3e^{x}$

$\left(c_1 e^{x} + 16 c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{6}{5} e^{x}\right) + 3\left(c_1 e^{x} - 4 c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{3}{5} e^{x}\right)$

$- 4\left(c_1 e^{x} + c_2 e^{-4x} + \frac{3}{5} x e^{x}\right) \stackrel{?}{=} 3e^{x}$

$c_1 e^{x} + 16 c_2 e^{-4x} + \frac{3}{5} x e^{x} + \frac{6}{5} e^{x} + 3 c_1 e^{x} - 12 c_2 e^{-4x} + \frac{9}{5} x e^{x} + \frac{9}{5} e^{x}$

$- 4 c_1 e^{x} - 4 c_2 e^{-4x} - \frac{12}{5} x e^{x} \stackrel{?}{=} 3e^{x}$

$\frac{6}{5} e^{x} + \frac{9}{5} e^{x} = \frac{15}{5} e^{x} \stackrel{\checkmark}{=} 3 e^{x}$

$y = c_1 e^{x} + c_2 e^{-4x} + \frac{3}{5} x e^{x}$ is a general solution of the differential Equation

(10) $y'' = 2y'$

2nd order or order is 2

$y = 3e^{2x}$ & $y = 2e^{2x} - 5$

$y' = 6e^{2x}$ | $y' = 4e^{2x}$

$y'' = 12e^{2x}$ | $y'' = 8e^{2x}$

$12e^{2x} \stackrel{\checkmark}{=} 2(6e^{2x})$

$y = 3e^{2x}$ is a particular solution for the D.E.

$8e^{2x} \stackrel{\checkmark}{=} 2(4e^{2x})$

$y = 2e^{2x} - 5$ is a particular solution for the D.E.