29.3 Partial Derivatives

If $z = f(x, y)$ and y is held constant, z becomes a function of x. The derivative of $f(x, y)$ with respect to x is called a partial derivative of z with respect to x, denoted $\frac{\partial z}{\partial x}$, $\frac{\partial f}{\partial x}$, or $f_x(x, y)$ $\quad \frac{\partial}{\partial x} f(x,y)$ or f_x

Similar The partial derivative of z with respect to y can be found and denoted, $\frac{\partial z}{\partial y}$, $\frac{\partial F}{\partial y}$, or $f_y(x, y)$.

p.897

③ $z = f(x,y) = 5x + 4x^2 y$

$f_x = 5 + 4y(2x) = 5 + 8xy$

$f_y = 0 + 4x^2 = 4x^2$

⑤ $f(x, y) = x e^{-2y}$

$f_x = e^{-2y}$

$f_y = x e^{-2y}(-2) = -2xe^{-2y}$

⑬ $z = f(x, y) = \sin x^2 y$

$f_x = \cos(x^2 y)\; y \cdot 2x = 2xy \cos(x^2 y)$

$f_y = \cos(x^2 y)\; x^2 = x^2 \cos(x^2 y)$

(15) $y = f(r,s) = \ln(r^2+6s)$

$$f_r = \frac{1}{r^2+6s}\, 2r = \frac{2r}{r^2+6s}$$

$$f_s = \frac{1}{r^2+6s}\, 6 = \frac{6}{r^2+6s}$$

(9) $\phi(r,s) = r\sqrt{1+2rs} = r(1+2rs)^{1/2}$

$$\phi_r = r\,\tfrac{1}{2}(1+2rs)^{-1/2}\, 2s + (1+2rs)^{1/2}(1) = \frac{rs}{(1+2rs)^{1/2}} + (1+2rs)^{1/2}$$

$$\phi_s = r\,\tfrac{1}{2}(1+2rs)^{-1/2}\, 2r = \frac{r^2}{\sqrt{1+2rs}}$$

The Possible 2nd Order Partial Derivatives are.

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = f_{xx}(x,y) \qquad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = f_{yy}(x,y)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = f_{yx}(x,y) \qquad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = f_{xy}(x,y)$$

(29) $z = f(x,y) = 2xy^3 - 3x^2y$

$f_x = 2y^3 - 6xy$ $\qquad$ $f_y = 6xy^2 - 3x^2$

$f_{xx} = -6y$ $\qquad$ $f_{yy} = 12xy$

$f_{xy} = 6y^2 - 6x$ $\qquad$ $f_{yx} = 6y^2 - 6x$

36

$P(A, v) = kAv^3$ k is a constant of proportionality.

$P_A = kv^3$

$P_v = kA(3v^2) = 3kAv^2$