28.7 Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int \underbrace{x}_{u}\,\underbrace{e^x\,dx}_{dv} \overset{IBP}{=} \overset{u}{x}\;\overset{v}{e^x} - \int \overset{v}{e^x}\,\overset{du}{dx} = xe^x - e^x + C$$

Let $u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = \int e^x dx$

$= e^x$

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(4) $\int \underbrace{x}_{u}\,\underbrace{\sin 2x\,dx}_{dv} \overset{IBP}{=} \overset{u}{x}\overset{v}{\left(-\frac{1}{2}\cos 2x\right)} - \int \overset{v}{-\frac{1}{2}\cos 2x}\,\overset{du}{dx}$

$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\left(\frac{1}{2}\right)\int \cos 2x(2)\,dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$

$u = x$ $\quad dv = \sin 2x\,dx$

$du = dx$ $\quad v = \frac{1}{2}\int \sin 2x \cdot 2\,dx$

$= \frac{-1}{2}\cos 2x$

(10) $\int \underbrace{\ln s}_{u}\,\underbrace{ds}_{dv} \overset{IBP}{=} \overset{u}{\ln s}\,\overset{v}{(s)} - \int \overset{v}{\cancel{s}}\,\overset{du}{\frac{1}{\cancel{s}}\,ds}$

$= s\ln s - \int ds$

$= s\ln s - s + C$

$u = \ln s$ $\quad dv = ds$

$du = \frac{1}{s}ds$ $\quad v = \int ds$

$= s$

(14) $\int 8x^2 \ln 4x \, dx$

$u = \ln 4x \qquad dv = 8x^2\,dx$

$du = \frac{4}{4x}\,dx = \frac{1}{x}\,dx \qquad v = 8\int x^2\,dx = 8\frac{x^3}{3}$

$$\overset{\text{IBP}}{=} \overset{u\ v}{\ln 4x\left(\tfrac{8}{3}x^3\right)} \overset{-}{-} \overset{\int v\,du}{\int \frac{8}{3}\cancel{x^3}^{\,2}\frac{1}{\cancel{x}}\,dx}$$

$$= \frac{8}{3}x^3\ln 4x - \frac{8}{3}\int x^2\,dx$$

$$= \frac{8}{3}x^3\ln 4x - \frac{8}{3}\left(\frac{1}{3}x^3\right) + C$$

$$= \frac{8}{3}x^3\ln 4x - \frac{8}{9}x^3 + C$$

(16) $\int_0^1 \underbrace{r^2}_{u}\,\underbrace{e^{2r}\,dr}_{dv}$

$u = r^2 \qquad dv = e^{2r}\,dr$

$du = 2r\,dr \qquad v = \frac{1}{2}\int e^{2r}\,2\,dr = \frac{1}{2}e^{2r}$

$$\overset{\text{IBP1}}{=} \overset{u\ v}{r^2\,\tfrac{1}{2}e^{2r}\Big|_0^1} \overset{-}{-} \overset{\int v\,du}{\int_0^1 \tfrac{1}{\cancel{2}}e^{2r}\,\cancel{2}r\,dr}$$

$$= \frac{1}{2}r^2e^{2r}\Big|_0^1 - \int_0^1 \underbrace{r}_{u}\,\underbrace{e^{2r}\,dr}_{dv}$$

$u = r \qquad dv = e^{2r}\,dr$

$du = dr \qquad v = \frac{1}{2}e^{2r}$

IBP2 $u\ v - \int v\,du$

$$= \frac{1}{2}r^2e^{2r}\Big|_0^1 - \left[r\,\frac{1}{2}e^{2r}\Big|_0^1 - \int_0^1 \frac{1}{2}e^{2r}\,dr\right]$$

$$= \left[\frac{1}{2}r^2e^{2r} - \frac{1}{2}re^{2r}\right]\Big|_0^1 - \frac{1}{2}\left(\frac{1}{2}\right)\int_0^1 e^{2r}(2)\,dr$$

$$= \frac{1}{2}r^2e^{2r} - \frac{1}{2}re^{2r} - \frac{1}{4}e^{2r}\Big|_0^1$$

$$= \left(\frac{1}{2}(1)^2e^{2(1)} - \frac{1}{2}\cdot 1\cdot e^{2(1)} - \frac{1}{4}e^{2(1)}\right) - \left(0 - 0 - \frac{1}{4}e^0\right)$$

$$= \cancel{\frac{1}{2}e^2} - \cancel{\frac{1}{2}e^2} - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4} - \frac{1}{4}e^2 \text{ or } \frac{1}{4}\left(1-e^2\right) \approx 1.597$$