28.6 Inverse Trigonometric Forms

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$$

a is a constant

p861

(4) $$\int \frac{dx}{\sqrt{49-x^2}} = \int \frac{du}{\sqrt{7^2-u^2}} = \sin^{-1}\frac{u}{7} + c = \sin^{-1}\frac{x}{7} + c$$

$a = \sqrt{49} = 7$
$u = x$
$du = dx$

(6) $$\int \frac{6p^2\,dp}{4+p^6} = 6\left(\frac{1}{3}\right)\int \frac{3p^2\,dp}{4+p^6} = 2\int \frac{du}{2^2+u^2} = 2\,\frac{1}{a}\tan^{-1}\frac{u}{a} + c$$

$$= 2\left(\frac{1}{2}\right)\tan^{-1}\frac{p^3}{2} + c$$

$$= \tan^{-1}\frac{p^3}{2} + c$$

$a = \sqrt{4} = 2$
$u = p^3 \Rightarrow u^2 = (p^3)^2 = p^6$
$du = 3p^2\,dp$

(18) $$\int \frac{\sec^2 x\,dx}{\sqrt{9-\tan^2 x}} = \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c$$

$$= \sin^{-1}\left(\frac{\tan x}{3}\right) + C$$

$a = \sqrt{9} = 3$
$u = \tan x$
$du = \sec^2 x\,dx$

(14) $$\int \frac{y\,dy}{4\sqrt{25-16y^2}} = \left(-\frac{1}{32}\right)\frac{1}{4}\int (25-16y^2)^{-1/2}(-32)y\,dy$$

$$= -\frac{1}{128}\int u^{-1/2}\,du$$

$$= -\frac{1}{128}\,\frac{u^{-1/2+1}}{-1/2+1} + C$$

$$= -\frac{1}{64}\sqrt{25-16y^2} + C$$

$a = 5$
$u = 4y$
$du = 4\,dy$ won't work with $y\,dy$

$u = 25-16y^2$
$du = -32y\,dy$

(20) $\int \frac{2\,dx}{x^2+8x+20}$

completing the square

$x^2+8x+20 = x^2+8x+16+4$

$\left(\frac{8}{2}\right)^2=16 \qquad = (x+4)^2+4$

$$=2\int \frac{dx}{4+(x+4)^2} = 2\int \frac{du}{a^2+u^2} = 2\,\frac{1}{a}\tan^{-1}\frac{u}{a}+C$$

$a=\sqrt{4}=2$

$u=x+4$

$du=dx$

$$=2\cdot\frac{1}{2}\tan^{-1}\left(\frac{x+4}{2}\right)+C$$

$$=\tan^{-1}\left(\frac{x+4}{2}\right)+C$$

$\int \frac{x}{1+x^2}\,dx$ natural log let $u=$ bottom

$\int \frac{x}{\sqrt{1+x^2}}\,dx$ general power rule let $u=$ inside parenthesis or radical

$\int \frac{dx}{1+x}$ natural log let $u=$ bottom