28.4 Basic Trigonometric Forms

From the derivatives of Trigonometric Functions we get

$$\boxed{\begin{array}{ll} \int \sin u \, du = -\cos u + C & \int \cos u \, du = \sin u + C \\ \int \sec^2 u \, du = \tan u + C & \int \csc^2 u \, du = -\cot u + C \\ \int \sec u \tan u \, du = \sec u + C & \int \csc u \cot u \, du = -\csc u + C \end{array}}$$

We saw on p. 846 #10 something similar to the following

$$\int \tan x \, dx = -\int \frac{-\sin x}{\cos x} dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -\int \frac{du}{u} = -\ln|u| + C = -\ln|\cos x| + C$$

$$\int \sec x \, dx = \int \frac{\sec x (\sec x + \tan x)}{\sec x + \tan x} dx$$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

or $\sec^2 x + \sec x \tan x \, dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\boxed{\begin{aligned} &\int \tan u \, du = -\ln|\cos u| + c \\ &\int \cot u \, du = \ln|\sin u| + c \\ &\int \sec u \, du = \ln|\sec u + \tan u| + c \\ &\int \csc u \, du = \ln|\csc u - \cot u| + c. \end{aligned}}$$

p.853

④ $(-1)\int 4 \sin(2-x)(-1)dx$

$u = 2-x$

$du = -dx$

$$= -4\int \sin u \, du = -4(-\cos u) + C = 4\cos(2-x) + C$$

⑥ $\frac{1}{2}\int 2\,4 \csc 8x \cot 8x \, dx$

$u = 8x$

$du = 8\,dx$

$$= \frac{1}{2}\int \csc u \cot u \, du = -\frac{1}{2}\csc u + C = -\frac{1}{2}\csc 8x + C$$

⑧ $\int \frac{\csc^2(e^{-x})}{e^x} dx = -1\int \csc^2 e^{-x}(-1 e^{-x} dx)$

$u = e^{-x}$

$du = -e^{-x}dx$

$$= -\int \csc^2 u \, du = -(-\cot u) + c = \cot(e^{-x}) + C$$

(14) $\int \frac{3\,dx}{\sin 4x} = 3\frac{1}{4}\int \csc 4x\, 4dx$ $\quad u = 4x$, $du = 4\,dx$

$= \frac{3}{4}\int \csc u\, du = \frac{3}{4}\ln|\csc u - \cot u| + C$

$= \frac{3}{4}\ln|\csc 4x - \cot 4x| + C$

(10) $\int_0^1 6\sin\frac{1}{2}t\,\sec\frac{1}{2}t\,dt = 6\int_0^1 \sin\frac{1}{2}t\,\frac{1}{\cos\frac{1}{2}t}\,dt$

$2\cdot 6\int_0^1 \tan\frac{1}{2}t\cdot\frac{1}{2}dt = 12\int_{t=0}^{t=1}\tan u\,du = 12(-\ln|\cos u|)\Big|_{t=0}^{t=1}$

$u = \frac{1}{2}t$, $du = \frac{1}{2}dt$

$= -12\ln|\cos\frac{1}{2}t|\Big|_0^1 = -12\ln\cos\frac{1}{2} - \underbrace{(-12\ln\cos 0)}_{0}$

$= -12\ln\cos\frac{1}{2} \approx 1.567$

(12) $\cos 2\theta = 2\cos^2\theta - 1$

$\int (2\cos^2 4x - 1)\,dx = \frac{1}{8}\int \cos 8x\cdot 8dx$ $\quad u = 8x$, $du = 8\,dx$

$= \frac{1}{8}\int \cos u\,du = \frac{1}{8}\sin u + C = \frac{1}{8}\sin 8x + C$

$\sin 2\theta = 2\sin\theta\cos\theta$

(18) $\int \frac{\sin 2x}{2\cos^2 x}dx = \int \frac{\cancel{2}\sin x\cos x}{\cancel{2}\cos^{\cancel{2}} x}d\theta = \tan x = -\ln|\cos x| + C$

(22) $\int \frac{1-\cot^2 x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x} - \frac{\cot^2 x}{\cos^2 x}\,dx$ $\quad \frac{\frac{\cos^2 x}{\sin^2 x}}{\cos^2 x} = \frac{1}{\sin^2 x}$

$= \int \sec^2 x - \csc^2 x\,dx = \tan x + \cot x + C$