28.2 Basic Logarithmic Form

$y = \ln u(x) \qquad y' = \frac{1}{u(x)} u'(x) \text{ or } \frac{u'(x)}{u(x)}$

Argument of log needs to be positive

$$\boxed{\int \frac{du}{u} = \int u^{-1}\,du = \ln|u| + c}$$

p. 846

(4) $-\frac{1}{4}\int \frac{-4\,dx}{1-4x}$

$u = 1-4x$

$du = -4\,dx$

$= -\frac{1}{4}\int \frac{du}{u} = -\frac{1}{4}\ln|u| + c = -\frac{1}{4}\ln|1-4x| + c$

(10) $\int \frac{9\sin 3x}{\cos 3x}\,dx = -\frac{1}{3}\cdot 9\int \underbrace{\frac{-3\sin 3x}{\cos 3x}\,dx}_{du}$

$u = \cos 3x$

$du = -3\sin 3x\,dx$

$= -3\int \frac{du}{u} = -3\ln|u| + c = -3\ln|\cos 3x| + c$

(14) $\int \frac{15e^{3x}}{1-e^{3x}}\,dx = -\frac{1}{3}\cdot 15\int \frac{(-3)e^{3x}}{1-e^{3x}}\,dx$

$u = 1-e^{3x}$

$du = -3e^{3x}\,dx$

$= -5\int \frac{du}{u} = -5\ln|u| + c = -5\ln|1-e^{3x}| + c$

⑧ $\int_{-1}^{3} \frac{8x^3}{x^4+1}\,dx = \frac{1}{4} 8 \int_{-1}^{3} \frac{4 \cdot x^3}{x^4+1}\,dx$

$u = x^4+1$
$du = 4x^3\,dx$

$= 2\int_{x=-1}^{x=3} \frac{du}{u} = 2 \ln|x^4+1| \Big|_{-1}^{3}$

$= 2\ln(3^4+1) - 2\ln((-1)^4+1)$

$= 2\ln 82 - 2\ln 2 \approx 7.43$

28.3 The Exponential Form

$y = e^{u(x)}$ $\qquad y' = e^{u(x)}\,u'(x)$

$$\boxed{\int e^u\,du = e^u + C}$$

p.849

④ $\int \underbrace{e^{x^4}}_{e^u} \underbrace{(4x^3\,dx)}_{du}$

$u = x^4$
$du = 4x^3\,dx$

$= \int e^u\,du = e^u + c = e^{x^4} + c$

⑩ $\int \frac{8x}{e^{x^2}}\,dx = -\frac{1}{2}\cdot 8 \int -2x\, e^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$= -4\int e^u\,du = -4e^u + c = -4e^{-x^2} + c$

⑭ $\int \sec^2 x\, e^{\tan x}\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \int e^u\,du = e^u + c = e^{\tan x} + c$

⑧ $\int_1^2 3e^{4x}\,dx = \frac{1}{4}3\int_1^2 4e^{4x}\,dx$

$u = 4x$

$du = 4\,dx$

$x = 1 \Rightarrow u = 4(1) = 4$

$x = 2 \Rightarrow u = 4(2) = 8$

$= \frac{3}{4}\int_{u=4}^{u=8} e^u\,du = \frac{3}{4}e^u \Big|_4^8 = \frac{3}{4}e^8 - \frac{3}{4}e^4$

≈ 2194.77