28.1 General Power Rule

$$\int x^2\,dx = \frac{1}{2+1}x^{2+1} + C = \frac{1}{3}x^3 + C$$

$$\int 4x + 2\,dx = 4\left(\frac{1}{2}x^2\right) + 2x + C = 2x^2 + 2x + C$$

$$\int \underbrace{(x^2-4x+3)^3}_{u^3}\,\frac{2}{2}\underbrace{(x-2)\,dx}_{du}$$

$u = x^2 - 4x + 3$

$du = 2x - 4\ dx$

$$= \frac{1}{2}\int u^3\,du = \frac{1}{2}\,\frac{1}{4}u^4 + C = \frac{1}{8}(x^2-4x+3)^4 + C$$

$$\frac{1}{(\quad)^3} = (\quad)^{-3}$$

$$\sqrt{(\quad)} = (\quad)^{1/2} \qquad \frac{1}{\sqrt{(\quad)}} = (\quad)^{-1/2}$$

$$\sqrt[3]{(\quad)} = (\quad)^{1/3} \qquad \frac{1}{\sqrt[3]{(\quad)}} = (\quad)^{-1/3}$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\left[\frac{1}{(\quad)} = (\quad)^{-1}\right]$$

p.842

④ $\int \underbrace{\cos^5 x}_{u^5} \underbrace{(-\sin x\, dx)}_{du}$

$u = \cos x$
$du = -\sin x\, dx$

$= \int u^5\, du = \frac{1}{6} u^6 + C = \frac{1}{6}\cos^6 x + C$

⑧ $\int \underbrace{\sec^3 x}_{u^3} \underbrace{(\sec x \tan x)\, dx}_{du}$

$u = \sec x$
$du = \sec x \tan x\, dx$

$= \int u^3\, du = \frac{1}{4} u^4 + C = \frac{1}{4}\sec^4 x + C$

⑭ $\int \frac{\sin^{-1} 4x}{\sqrt{1-16x^2}}\, dx$

$u = \sin^{-1} 4x$
$du = \frac{1}{\sqrt{1-(4x)^2}}\, 4\, dx = \frac{4}{\sqrt{1-16x^2}}\, dx$

$= \frac{1}{4}\int \underbrace{\sin^{-1} 4x}_{u} \cdot \underbrace{\frac{4}{\sqrt{1-16x^2}}\, dx}_{du}$

$= \frac{1}{4}\int u\, du = \frac{1}{4}\cdot\frac{1}{2} u^2 + C = \frac{1}{8}\left[\sin^{-1} 4x\right]^2 + C$

(18) $\displaystyle\int_1^e \frac{(1-2\ln x)}{4x}\,dx$

$u = 1 - 2\ln x$

$du = -2\frac{1}{x}\,dx = -\frac{2}{x}\,dx$

$$\left(-\frac{1}{2}\right)\frac{1}{4}\int_1^e \underbrace{(1-2\ln x)}_{u}\,\underbrace{-2\frac{1}{x}\,dx}_{du}$$

$$= -\frac{1}{8}\int_{x=1}^{x=e} u\,du = -\frac{1}{8}\,\frac{1}{2}u^2\Big|_{x=1}^{x=e}$$

$$= -\frac{1}{16}(1-2\ln x)^2\Big|_{x=1}^{x=e} = -\frac{1}{16}(1-2\overbrace{\ln e}^{1})^2 - \left[-\frac{1}{16}(1-2\overbrace{\ln 1}^{0})^2\right]$$

$$= -\frac{1}{16}(1) - \left(-\frac{1}{16}\right)(1) = 0$$