27.7 L'Hospital's Rule

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 2+2 = 4$$

Indeterminite Form: When Limit gives $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hospitall's Rule

If $u(x)$ & $v(x)$ are differentiable such that $v'(x) \neq 0$

and if $\lim_{x\to a} u(x) = 0$ and $\lim_{x\to a} v(x) = 0$

or $\lim_{x\to a} u(x) = \infty$ and $\lim_{x\to a} v(x) = \infty$

then $$\lim_{x\to a} \frac{u(x)}{v(x)} = \lim_{x\to a} \frac{u'(x)}{v'(x)}$$

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(6) $\lim_{x\to\infty} \frac{e^x}{x^2}$ $\quad \lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to\infty} x^2 = \infty$

Indeterminate form since $\frac{\infty}{\infty}$.

$$\lim_{x\to\infty} \frac{e^x}{x^2} \overset{LR}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(x^2)} = \lim_{x\to\infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{e^x}{2x} \overset{LR}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(e^x)}{\frac{d}{dx}(2x)} = \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

Indeterminate Forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$

$$\frac{\infty}{\#} = \infty \qquad \frac{\#}{\infty} = 0 \qquad \frac{-\infty}{\#} = -\infty \qquad \frac{\#}{-\infty} = 0$$

⑧ $\lim_{x\to 1} \frac{\ln x}{x-1}$ $\lim_{x\to 1} \ln x = 0$ $\lim_{x\to 1} x-1 = 0$

Indeterminate form

$$\lim_{x\to 1} \frac{\ln x}{x-1} \overset{\text{L.R.}}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = \lim_{x\to 1} \frac{1}{x} = 1$$

⑩ $\lim_{x\to 0} \frac{\ln \cos x}{x}$ $\lim_{x\to 0} \ln \cos x = 0$ & $\lim_{x\to 0} x = 0$

Indeterminate Form

$$\lim_{x\to 0} \frac{\ln \cos x}{x} \overset{\text{LR}}{=} \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{1} = \lim_{x\to 0} -\frac{\sin x}{\cos x} = -\frac{0}{1} = 0$$

⑭ $$\lim_{x\to\infty} \frac{x^2+x}{e^x+1} \overset{\text{LR}}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(x^2+x)}{\frac{d}{dx}(e^x+1)} = \lim_{x\to\infty} \frac{2x+1}{e^x}$$

($x^2+x \to \infty$, $e^x+1 \to \infty$; $2x+1 \to \infty$, $e^x \to \infty$)

$$\overset{\text{LR}}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(2x+1)}{\frac{d}{dx}(e^x)} = \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$

⑱ $$\lim_{z\to\infty} z e^{-z} = \lim_{z\to\infty} \frac{z}{e^z} \overset{\text{L.R.}}{=} \lim_{z\to\infty} \frac{1}{e^z} = 0$$

($z \to \infty$, $e^{-z} \to 0$; $e^z \to \infty$)

Indeterminate Form $0 \cdot \infty$

$\frac{6}{3} = 2$

$\frac{0}{2} = 0$ $\frac{0}{0} = ?$ $\frac{2}{0} =$ undefined.