

27.5 Derivatives of Logarithmic Functions

$$y = \log_b x$$

Natural logarithm: $y = \ln x = \log_e x$ $e \approx 2.718\dots$

$$y = \ln u(x) \Rightarrow y' = \frac{1}{u(x)} \cdot u'(x) = \frac{u'(x)}{u(x)}$$

Examples

$$\begin{aligned} 1. \quad y = \ln(3x^2 + 4x) &\Rightarrow y' = \frac{1}{3x^2 + 4x} (3x^2 + 4x)' \\ &= \frac{6x + 4}{3x^2 + 4x} \end{aligned}$$

$$\begin{aligned} 2. \quad y = \ln(\underbrace{\sin 3x}_{u(x)}) &\Rightarrow y' = \frac{1}{\sin 3x} (\sin 3x)' = \frac{3 \cos 3x}{\sin 3x} \\ &= 3 \cot 3x \end{aligned}$$

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$$y = \ln(4x - 3)^3$$

$$y' = \frac{1}{(4x-3)^3} [(4x-3)^3]' = \frac{3(4x-3)^2 \cdot 4}{(4x-3)^3} = \frac{12}{4x-3}$$

$$4. (16) y = \underbrace{6x^2}_F \underbrace{\ln 5x}_S$$

$$y' = 6x^2 [\ln 5x]' + \ln 5x [6x^2]'$$

$$= 6x^2 \cdot \frac{1}{5x} \cdot 5 + (\ln 5x) 12x = 6x + 12x \ln 5x$$

$$5. y = \tan^{-1}(\ln 2x)$$

$$y' = \frac{1}{1+(\ln 2x)^2} (\ln 2x)' = \frac{1}{1+(\ln 2x)^2} \cdot \frac{2}{2x} = \frac{1}{x(1+(\ln 2x)^2)}$$

27.6 Derivatives of Exponential Functions

$$f(x) = b^x \quad g(x) = e^x$$

$$y = e^{u(x)} \Rightarrow y' = e^{u(x)} u'(x) \text{ or } u'(x) e^{u(x)}$$

Examples

$$1. y = 3e^{x^2} \Rightarrow y' = 3e^{x^2} (x^2)' = 3e^{x^2} 2x = 6xe^{x^2}$$

$$2. y = 4e^{x^3+2x} \Rightarrow y' = 4e^{x^3+2x} (3x^2+2) = (12x^2+8)e^{x^3+2x}$$

$$3. y = e^{\cos(3x)} \Rightarrow y' = e^{\cos 3x} (-\sin 3x \cdot 3) = -3 \sin 3x e^{\cos 3x}$$

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⑭

$$u = \frac{e^{0.5v} \leftarrow T}{2v \leftarrow B}$$

$$u' = \frac{2v e^{.5v} (.5) - e^{.5v} (2)}{(2v)^2}$$

$$\leftarrow \frac{2v(e^{.5v})' - e^{.5v}(2v)'}{(2v)^2}$$

$$= \frac{ve^{.5v} - 2e^{.5v}}{4v^2} \text{ or } \frac{e^{.5v}(v-2)}{4v^2}$$

$$5. \textcircled{8} \quad y = 0.6 \ln(e^{5x} + 3^4) \text{ or } 0.6 \ln(e^{5x} + 81)$$

$$y' = 0.6 \frac{1}{e^{5x} + 81} [e^{5x} + 81]' = 0.6 \frac{e^{5x} \cdot 5}{e^{5x} + 81} = \frac{3e^{5x}}{e^{5x} + 81}$$

$$6. \textcircled{30} \quad y = 6 \tan e^{x+1}$$

$$y' = 6 \sec^2(e^{x+1}) [e^{x+1}]' = 6 \sec^2(e^{x+1}) \cdot e^{x+1} \quad (1)$$

$$= 6e^{x+1} \sec^2(e^{x+1})$$

General Forms

$$y = \log_b u(x) \Rightarrow y' = \frac{1}{u(x)} \cdot u'(x) \log_b e$$

Example

$$y = \log_4 (x^2+3) \Rightarrow y' = \frac{1}{x^2+3} 2x \log_4 e \quad \text{or} \quad \frac{2x \log_4 e}{x^2+3}$$

$$y = b^{u(x)} \Rightarrow y' = b^{u(x)} u'(x) \ln b$$

Example

$$y = 3^{\sin x} \Rightarrow y' = 3^{\sin x} \cos x \ln 3 \quad \text{or} \quad \ln 3 \cos x 3^{\sin x}$$