27.3 Derivatives of Inverse Trigonometric Functions

$y = \sin^{-1} u \Rightarrow \sin y = u$ or $u = \sin y$

$$\frac{du}{dx} = \cos y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} \frac{du}{dx}$$

$$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 y}} \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2} \frac{du}{dx}$$

Example

$$y = \sin^{-1}(\overset{u}{\overset{\downarrow}{2x}}) \Rightarrow y' = \frac{1}{\sqrt{1-(2x)^2}}(2x)' = \frac{2}{\sqrt{1-4x^2}}$$

$$y = \tan^{-1}(\overset{u}{3x^2}) \Rightarrow y' = \frac{1}{1+(3x^2)^2}(3x^2)' = \frac{6x}{1+9x^4}$$

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(4) $R = 3\sin^{-1}(\overbrace{4-t^2}^{u})$ $\quad (4-t^2)(4-t^2)$

$$R' = 3\left(\frac{1}{\sqrt{1-(4-t^2)^2}}\right)(4-t^2)' = \frac{3(-2t)}{\sqrt{1-(16-8t^2+t^4)}} = \frac{-6t}{\sqrt{-15+8t^2-t^4}}$$

Like (6) $y = \sin^{-1}\overbrace{\sqrt{1-2x}}^{u}$

$$y' = \frac{1}{\sqrt{1-(\sqrt{1-2x})^2}}\left[(1-2x)^{1/2}\right]' = \frac{1}{\sqrt{1-(1-2x)}}\,\frac{1}{\cancel{2}}(1-2x)^{-1/2}(-\cancel{2})$$

$$= \frac{-1}{\sqrt{2x}\sqrt{1-2x}} = \frac{-1}{\sqrt{2x-4x^2}}$$

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(8) $\theta = .2\cos^{-1}(5t)$

$$\theta' = .2\left(-\frac{1}{\sqrt{1-(5t)^2}}\right)5 = -\frac{1}{\sqrt{1-25t^2}}$$

(12) $y = \tan^{-1}(1-2x)$

$$y' = \frac{1}{1+(1-2x)^2}(-2) = \frac{-2}{1+1-4x+4x^2} = \frac{-2}{4x^2-4x+2} = \frac{-1}{2x^2-2x+1}$$

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old (16) $y = (\overset{F}{x^2})(\overset{S}{\cos^{-1}x})$ $\quad y' = FS' + SF'$

$$y' = x^2\left(-\frac{1}{\sqrt{1-x^2}}\right) + (\cos^{-1}x)\,2x = \frac{-x^2}{\sqrt{1-x^2}} + 2x\cos^{-1}x$$

(42) Line Normal to $y = 2\sin^{-1}(0.5x)$ is parallel to line $y = 1 - x$

$m = -1$

$$m_{tan} = y' = 2\frac{1}{\sqrt{1-(.5x)^2}}(.5) = \frac{1}{\sqrt{1-.25x^2}}$$

Normal is perpindicular to tangent.

$$m_{normal} = -\sqrt{1-.25x^2} = -1$$

$$\left(\sqrt{1-.25x^2}\right)^2 = (1)^2$$

$$1 - .25x^2 = 1$$

$$-1 \qquad -1$$

$$-.25x^2 = 0$$

$$x = 0$$