27.1 Derivatives of the Sine & Cosine Functions

$$f'(x) = \frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$y = \sin x$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0} \frac{\sin x\cosh + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\left[\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

$$= \sin x(0) + \cos x(1)$$

$$= \cos x$$

$y = \sin u \Rightarrow \frac{dy}{dx} = \cos u \frac{du}{dx}$ or $(\cos u)\, u'$

$y = \cos u \Rightarrow \frac{dy}{dx} = -\sin u \frac{du}{dx}$ or $-(\sin u)\, u'$

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(4) $y = 3\sin 4x \Rightarrow y' = 3(\sin 4x)' = 3\cos 4x\,(4x)'$

$= 3(\cancel{\cos}\ 4x)4 = 12\cos 4x$

(8) $y = \cancel{\cos}(1-2x) \Rightarrow y' = [-\sin(1-2x)](1-2x)' = -[\sin(1-2x)](-2)$

$= 2\sin(1-2x)$

(10) $y = x^2 - \cos(1-3x) \Rightarrow y' = 2x - [-\sin(1-3x)](-3)$

$= 2x - 3\sin(1-3x)$

$$\left([u(x)]^n\right)' = n[u(x)]^{n-1}u'(x)$$

(12) $y = 3\sin^3(2x^4+1) \Rightarrow y' = 3\left[3\sin^2(2x^4+1)\right]\left(\sin(2x^4+1)\right)'$

$$= 9\sin^2(2x^4+1)\cos(2x^4+1)\cdot 8x^3$$

$$= 72x^3\sin^2(2x^4+1)\cos(2x^4+1)$$

(14) $y = 4\cos^2\sqrt{x} = 4\left[\cos x^{1/2}\right]^2$

$y' = 4\cdot 2\left[\cos x^{1/2}\right]^{2-1}\left(\cos x^{1/2}\right)'$

$= 8\cos x^{1/2}\left(-\sin x^{1/2}\right)\frac{1}{2}x^{-1/2}$

$= -4\dfrac{\cos\sqrt{x}\,\sin\sqrt{x}}{\sqrt{x}}$

$x^{-1/2} = \dfrac{1}{x^{1/2}} = \dfrac{1}{\sqrt{x}}$

(16) $V = \underbrace{6t^2}_{f}\,\underbrace{\sin 3\pi t}_{S}$ $\qquad V = f\cdot S \Rightarrow V' = fS' + Sf'$

$V = 6t^2[\sin 3\pi t]' + \sin 3\pi t\,[6t^2]'$

$= 6t^2(\cos 3\pi t)3\pi + \sin 3\pi t\,(12t)$

$= 18\pi t^2\cos 3\pi t + 12t\cdot\sin 3\pi t$

(24) $T = \dfrac{4z+3 \leftarrow t}{\sin\pi z \leftarrow B}$ $\qquad T' = \dfrac{Bt' - tB'}{B^2}$

$T' = \dfrac{(\sin\pi z)4 - (4z+3)\cos\pi z\cdot\pi}{\sin^2\pi z}$

$= \dfrac{4\sin\pi z - (4z+3)\pi\cos\pi z}{\sin^2\pi z}$

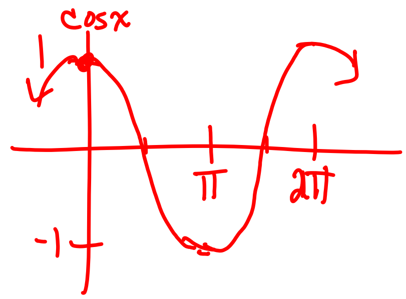
(48) a) $y = x + \sin x$

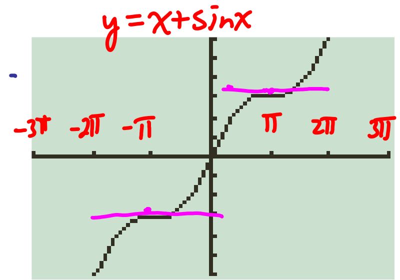
$m_{tan} = y' = 1 + \cos x$

horizontal tangent means $m_{tan} = 0$

$1 + \cos x = 0 \Rightarrow \cos x = -1$

$x = \ldots -3\pi, -\pi, \pi, 3\pi, 5\pi, \ldots$





(20) $y = \underbrace{\sin 3x}_{f}\ \underbrace{\cos 4x}_{s}$

$y' = \sin 3x\,(\cos 4x)' + \cos 4x\,(\sin 3x)'$

$= \sin 3x(-4)\sin 4x + \cos 4x\,(3\cos 3x)$

$= 3\cos 3x\,\cos 4x - 4\sin 3x\,\sin 4x$

(30) $z = .2\cos(4\sin 3\phi)$

$z' = .2(-\sin(4\sin 3\phi))(4\cos 3\phi)3$

$= -2.4\sin(4\sin 3\phi)(\cos 3\phi)$

(52) $d = 2.5\cos 16t$

$v = d' = 2.5 \times 16(-\sin 16t) = -40\sin 16t$

$a = d'' = -40 \times 16(\cos 16t) = -640\cos 16t$

$a\big|_{t=1.5} = -640\cos(16 \times 1.5) = -271\ m/s^2$