

Take your time and make sure you follow all instructions. Where necessary, work must be shown in order to receive partial credit.

Find the derivatives of the following functions.

[10 pts each]

1. $y = \cos(6x^3)$

$$y' = -\sin(6x^3) \cdot 3x^2 = -3x^2 \sin(6x^3)$$

2. $y = \sin^{-1}(5x^2)$

$$y' = \frac{1}{\sqrt{1-(5x^2)^2}} \cdot 10x = \frac{10x}{\sqrt{1-25x^4}}$$

3. $y = \ln(x^3 - x + 1)$

$$y' = \frac{3x^2 - 1}{x^3 - x + 1}$$

Find the derivatives of the following function.

4. $y = e^{\csc 4x^2}$

$$y' = e^{\csc 4x^2} - \csc 4x^2 \cot 4x^2 \cdot 8x$$

$$= -8x \csc 4x^2 \cot 4x^2 e^{\csc 4x^2}$$

Evaluate the limit (if it exists) using L'Hospital's Rule.

[12 points]

5. $\lim_{x \rightarrow 2} \frac{3 \sin(x-2)}{x-2}$ $\begin{matrix} \nearrow 0 \\ \searrow 0 \end{matrix}$

$$\stackrel{LR}{=} \lim_{x \rightarrow 2} \frac{3 \cos(x-2)}{1} = 3$$

Find the derivatives of the following functions.

6. $y = \sec^4(e^{2x^3})$

$$y' = 4 \sec^3(e^{2x^3}) \sec(e^{2x^3}) \tan(e^{2x^3}) e^{2x^3} 6x^2$$

$$= 24x^2 e^{2x^3} \sec^4(e^{2x^3}) \tan(e^{2x^3})$$

7. $y = \underbrace{4x^3}_F \ln(\underbrace{3x^2}_S)$

$$y' = 4x^3 \frac{6x}{3x^2} + \ln(3x^2) 12x^2$$

$$= 8x^2 + 12x^2 \ln(3x^2)$$

Evaluate the following integrals.

8. $\int \tan^5 3x \sec^2 3x \, dx$

$$u = \tan 3x$$

$$du = 3 \sec^2 3x \, dx$$

$$\frac{1}{3} \int \tan^5 3x \cdot 3 \sec^2 3x \, dx$$

$$= \frac{1}{3} \int u^5 \, du = \frac{1}{3} \frac{u^6}{6} + C = \frac{1}{18} \tan^6 3x + C$$

9. $\int \frac{e^{4t}}{(5+2e^{4t})^3} \, dt$

$$u = 5 + 2e^{4t}$$

$$du = 8e^{4t} \, dt$$

$$= \frac{1}{8} \int (5+2e^{4t})^{-3} \cdot 8e^{4t} \, dt = \frac{1}{8} \int u^{-3} \, du$$

$$= \frac{1}{8} \frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{16(5+2e^{4t})^2} + C$$