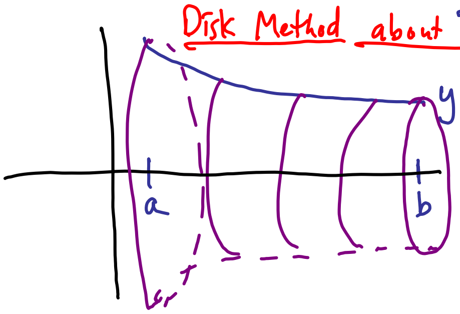
26.3 Volumes by Integration

Disk Method about the x-axis



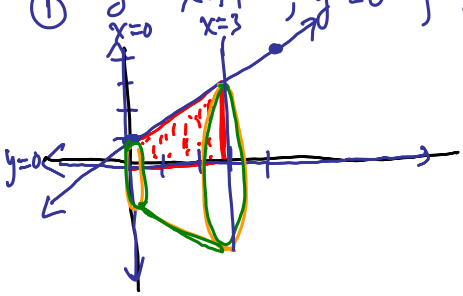
Area of Circle is $A = \pi r^2$

$$V = \int_a^b \pi [f(x)]^2 dx$$

$$= \pi \int_a^b [f(x)]^2 dx$$

Find the volume generated by revolving the regions bounded by the given curves about the x-axis.

Example ① $y = x+1$, $y=0$, $x=3$, $x=0$



$$V = \pi \int_a^b [f(x)]^2 dx$$

$$V = \pi \int_0^3 (x+1)^2 dx = \pi \int_0^3 x^2 + 2x + 1 \, dx$$

$$= \pi \left(\frac{x^3}{3} + \frac{2x^2}{2} + x \right) \Big|_0^3$$

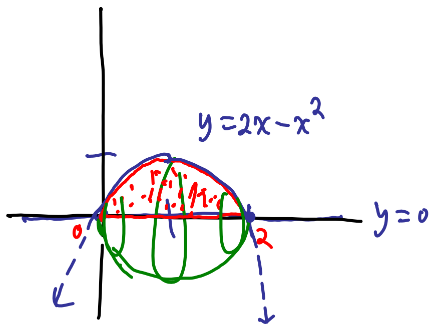
$$= \pi \left[\left(\frac{27}{3} + 9 + 3 \right) - 0 \right]$$

$$= 21\pi$$

p. 774

10 $\quad y = 2x - x^2, \quad y = 0$

$(2x-x^2)(2x-x^2)$

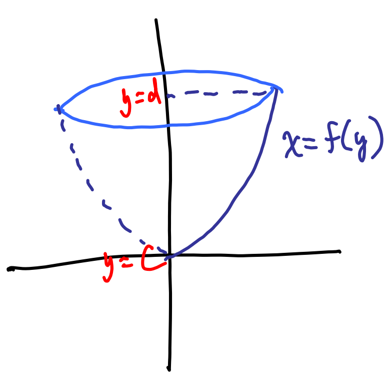


$$V = \pi \int_a^b [f(x)]^2\, dx$$
$$= \pi \int_0^2 (2x - x^2)^2\, dx$$
$$= \pi \int_0^2 x^4 - 4x^3 + 4x^2\, dx$$
$$= \pi \left(\frac{x^5}{5} - \frac{\cancel{4}x^4}{\cancel{4}} + \frac{4x^3}{3} \right) \Big|_0^2$$
$$= \pi \left[\left(\frac{2^5}{5} - 2^4 + \frac{4}{3}(2^3) \right) - 0 \right]$$
$$= \pi \left[\frac{32}{5} - 16 + \frac{32}{3} \right] = \frac{96 - 240 + 160}{15} \pi$$
$$= \frac{16}{15} \pi$$

See Example 1 p. 770

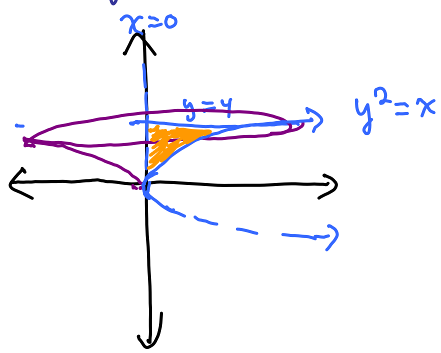
p. 774: 3, 7, 9, 13

Disk method about the y-axis



$$V = \pi \int_c^d [f(y)]^2\, dy.$$

(20) $y^2 = x \quad y = 4 \quad x = 0$



$y^2 = x$
$y = \pm\sqrt{x}$

$$V = \pi \int_{y=c}^{y=d} [f(y)]^2 \, dy$$

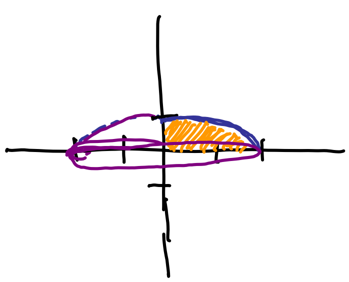
$$V = \pi \int_0^4 [y^2]^2 \, dy = \pi \int y^4 \, dy$$

$$= \pi \frac{y^5}{5} \Big|_0^4$$

$$= \pi \frac{4^5}{5} = \frac{1024}{5}\pi$$

(24) $x^2 + 4y^2 = 4$ ⟶ solve for y to graph

$4y^2 = 4 - x^2$
$y^2 = \frac{4 - x^2}{4}$
$y = \pm\sqrt{\frac{4 - x^2}{4}}$



Solve for x:
$x^2 = 4 - 4y^2$
$x = \pm\sqrt{4 - 4y^2}$

$$V = \pi \int_0^1 \left(\sqrt{4 - 4y^2}\right)^2 dy = \pi \int_0^1 4 - 4y^2 \, dy$$

$$= \pi \left[4y - \frac{4y^3}{3}\right]\Big|_0^1$$

$$= \pi \left[4 - \frac{4}{3}\right] = \frac{8\pi}{3}$$

p. 774-775: 4, 17, 19, 23