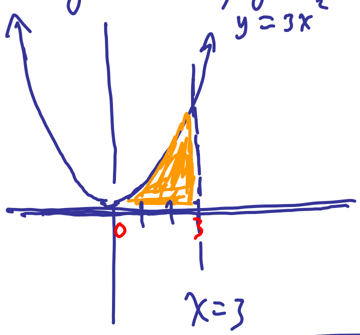
26.2 Areas By Integration

p. 769

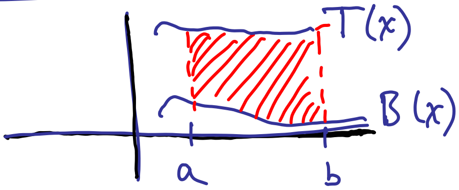
4) $y = 3x^2$; $y = 0$; $x = 3$



$$A = \int_0^3 3x^2\,dx$$

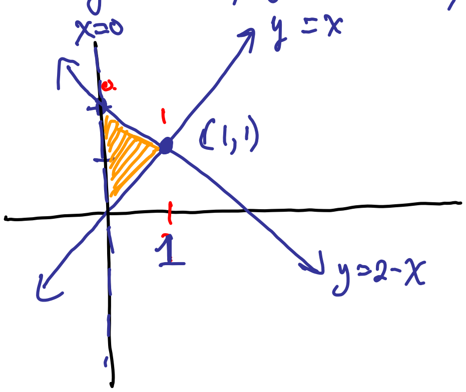
$$= \frac{3x^3}{3}\Big|_0^3 = x^3\Big|_0^3$$

$$= 3^3 - 0^3 = \boxed{27}$$



$$A = \int_a^b T(x) - B(x)\,dx$$

16) $y = x$; $y = 2 - x$; $x = 0$



Find Intersection (set equal)

$$x = 2 - x$$
$$+x \quad +x$$
$$2x = 2$$
$$x = 1$$

$$A = \int_0^1 \underbrace{2 - x}_{T(x)} - \underbrace{x}_{B(x)}\,dx$$

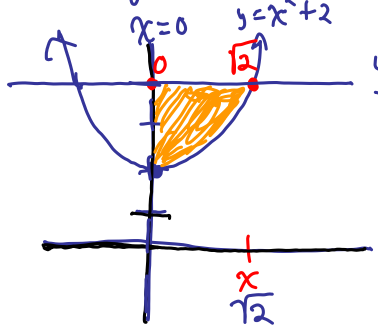
$$= \int_0^1 2 - 2x\,dx$$

$$= 2x - \frac{2x^2}{2}\Big|_0^1 = 2x - x^2\Big|_0^1$$

$$= (2(1) - 1^2) - (2(0) - 0^2) = 1 - 0$$

$$= 1$$

(6) $y = x^2+2$; $x=0$; $y=4$ $(x>0)$



Find Intersection between $y=x^2+2$ + $y=4$

$$x^2+2=4$$
$$x^2=2 \Rightarrow x=\pm\sqrt{2}$$

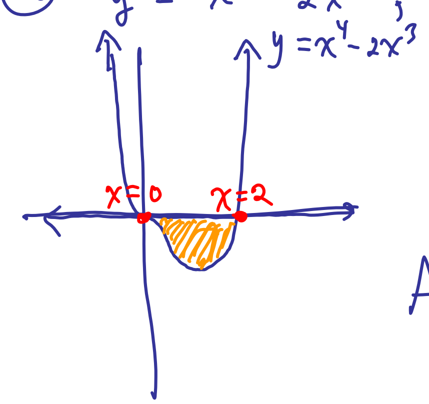
$$A=\int_0^{\sqrt{2}} \overset{T(x)}{4} - \overset{B(x)}{(x^2+2)}\,dx = \int_0^{2} 4-x^2-2\,dx$$

$$=\int_0^{\sqrt{2}} 2-x^2\,dx$$

$$= 2x-\frac{x^3}{3}\Big|_0^{\sqrt{2}} = \left(2\sqrt{2}-\frac{(\sqrt{2})^3}{3}\right)-0$$

$$= 2\sqrt{2}-\frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3} \text{ or } 1.886$$

(18) $y = x^4-2x^3$; $y=0$



Find Intersection.

$$x^4-2x^3=0$$
$$x^3(x-2)=0$$
$$x^3=0 \qquad x-2=0$$
$$x=0 \qquad x=2$$

$$A=\int_0^2 0-(x^4-2x^3)\,dx = \int_0^2 -x^4+2x^3\,dx$$

$$= -\frac{x^5}{5}+2\frac{x^4}{4}\Big|_0^2 = -\frac{x^5}{5}+\frac{x^4}{2}\Big|_0^2$$

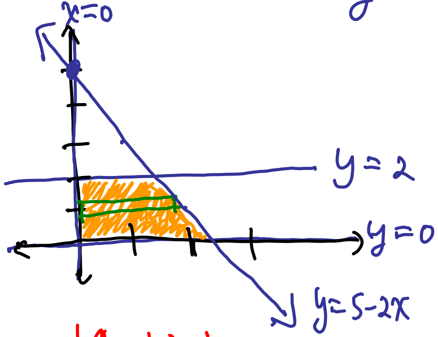
$$= -\frac{2^5}{5}+\frac{2^4}{2}-0 = -\frac{32}{5}+\frac{16}{2} = -\frac{32}{5}+8$$

$$= \frac{8}{5} \text{ or } 1.6$$

p.764: 3, 7, 17, 19, 23

Using Horizontal Areas.

Area bounded by $y = 5-2x$, $x=0$, $y=0$, $y=2$



$\frac{1}{2}(b_1+b_2)\, h$

$A = \frac{1}{2}(2.5+1.5)\,2$

$= 4$

Solve $y = 5-2x$ for x

$$\frac{y-5}{-2} = \frac{-2x}{-2}$$

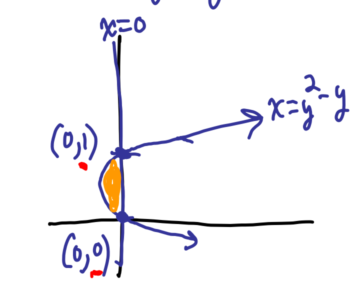
$$x = -\frac{y}{2} + \frac{5}{2}$$

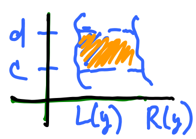
$$\int_{y=0}^{y=2} -\frac{y}{2} + \frac{5}{2}\, dy$$

$$= -\frac{1}{2}\frac{y^2}{2} + \frac{5}{2}y \Big|_0^2$$

$$= -\frac{1}{4}(2)^2 + \frac{5}{2}(2) - 0 = -1+5 = \boxed{4}$$

(14) $x = y^2 - y$, $x=0$





$$A = \int_c^d R(y) - L(y)\, dy$$

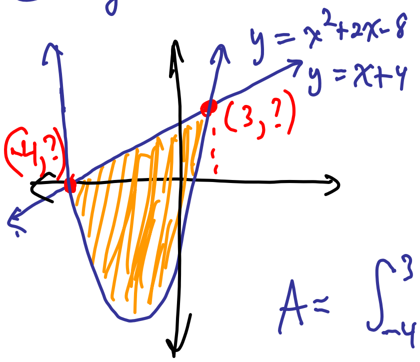
$$A = \int_0^1 \underset{R(y)}{0} - \underset{L(y)}{(y^2-y)}\, dy$$

$$= \int_0^1 -y^2 + y\, dy$$

$$= \frac{-y^3}{3} + \frac{y^2}{2}\Big|_0^1 = \frac{-(1)^3}{3} + \frac{1^2}{2} - 0$$

$$= -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$

(26) $y = x^2 + 2x - 8$, $y = x + 4$



Find intersections

$$x^2 + 2x - 8 = x + 4$$
$$\quad -x \;\; -4 \quad -x \;\; -4$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4 \text{ or } 3$$

$$A = \int_{-4}^{3} \overset{T(x)}{(x+4)} - \overset{B(x)}{(x^2 + 2x - 8)}\, dx$$

$$= \int_{-4}^{3} x + 4 - x^2 - 2x + 8 \, dx = \int_{-4}^{3} -x^2 - x + 12 \, dx$$

$$= -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 12x \Big|_{-4}^{3} \qquad \approx 57.1\overline{6}$$

$$= \left[-\frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 + 12(3)\right] - \left[-\frac{1}{3}(-4)^3 - \frac{1}{2}(-4)^2 + 12(-4)\right] = \frac{343}{6}$$

p. 769: 3, 7, 17, 19, 23 , 5, 9, 15,