26.1 Applications of the Indefinite Integral

$s(t)$ = displacement at time t.

$v(t) = s'(t) =$ velocity at time t.

$a(t) = s''(t) = v'(t) =$ acceleration at time t.

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(6) $V = 6t - 6t^2$

$$\frac{ds}{dt} = V$$
$$\int ds = \int v\,dt$$
$$s = \int 6t - 6t^2\,dt$$
$$= 6\frac{t^2}{2} - 6\frac{t^3}{3} + c$$
$$s = 3t^2 - 2t^3 + c$$

Initial Displacement

$t=0$ gives $s=0$

$S(0)=0$

Find c use $s=0$ when $t=0$

$$0 = 3(0)^2 - 2(0)^3 + c$$
$$0 = 0 + c$$
$$c = 0$$

$$\boxed{\begin{array}{l} s(t) = 3t^2 - 2t^3 \\ s(.75) = 3(.75)^2 - 2(.75)^3 = .84 \text{ mm} \end{array}}$$

(4) $V = 16$ ft/s when $t=0$

$$a = \frac{dv}{dt} = -5\text{ ft/s}^2$$
$$\frac{dv}{dt} = a$$
$$\int dv = \int a\,dt$$
$$V = \int a\,dt$$
$$v = \int -5\,dt$$
$$v = -5t + c$$

Find c: $V=16$ when $t=0$

$$16 = -5(0) + c$$
$$c = 16$$

$$V(t) = -5t + 16$$

at 6.0 s $V(6) = -5(6) + 16 = -14.0$ ft/s

p. 763: 1, 3, 5, 7

(8) $a = \dfrac{600t}{(60+0.5t^2)^2}$ $\quad V=0$ when $t=0$

$\dfrac{dV}{dt} = a$

$u = 60 + .5t^2$

$du = .5(2t)\,dt = t\,dt$

$V = \int a\,dt$

$= \int 600\underbrace{t\,(60+.5t^2)^{-2}\,dt}_{du}$

$= 600\int u^{-2}\,du$

$= 600\,\dfrac{u^{-1}}{-1} + c = -\dfrac{600}{u} + c$

$$\boxed{V = -\frac{600}{60+.5t^2} + c}$$

Find c

$V=0$ when $t=0$

$0 = \dfrac{-600}{60+.5(0)^2} + c$

$0 = -10 + c$

$c = 10$

$$\boxed{V(t) = \frac{-600}{60+.5t^2} + 10}$$

(10) $a = -1.6$ $\quad$ When $t=0$, $V = -2.0$ m/s, $s = 5.0$ m

$V = \int a\,dt = \int -1.6\,dt$

$V(t) = -1.6t + c_1$

Find c_1 use $V=-2.0$ when $t=0$

$-2.0 = -1.6(0) + c_1$

$c_1 = -2.0$

$V(t) = -1.6t - 2.0$

$S = \int v\,dt = \int -1.6t - 2.0\,dt$

$S(t) = -1.6\dfrac{t^2}{2} - 2.0t + c_2$

$= -.8t^2 - 2.0t + c_2$

Find c_2 use $S = 5.0$ when $t=0$

$5.0 = -.8(0)^2 - 2.0(0) + c_2 \Rightarrow c_2 = 5.0$

$S(t) = -.8t^2 - 2.0t + 5.0$

Reaches ground when $S=0$

$0 = -.8t^2 - 2.0t + 5.0$

$t = 1.55$ sec

$V(1.55) = -1.6(1.55) - 2.0$

$= -4.48$ m/s

velocity is 4.48 m/s when Lander hits the ground.

(18) q = charge at time t.
i = current at time t.

the current i equals the time rate of change of the charge.

$$i = \frac{dq}{dt} \Rightarrow q = \int i\,dt$$

V_c = voltage across a capacitor with capacitance C.

$$V_c = \frac{q}{C} = \frac{1}{C}\int i\,dt$$

(18) $i = .3 - .2t$

$$q = \int i\,dt = \int .3 - .2t\,dt$$
$$= .3t - .2\frac{t^2}{2} + c$$
$$q = .3t - .1t^2 + c$$

$q = 0$ when $t = 0$

Find c

$$0 = .3(0) - .1(0)^2 + c \Rightarrow c = 0$$
$$q(t) = .3t - .1t^2$$
$$q(.050) = .3(.050) - .1(.050)^2 = \boxed{.0147\,C}$$

θ = position of angle at time t = angular displacement. s

ω = angular velocity $= \frac{d\theta}{dt}$ $\qquad v = \frac{ds}{dt}$

Little omega

α = angular acceleration $= \frac{d^2\theta}{dt^2}$ or $\frac{d\omega}{dt}$ $\qquad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Little alpha

$$\omega = \int \alpha\,dt \quad \text{and} \quad \theta = \int \omega\,dt$$

p.763–764: 1–7 odd, 9, 15, 19, 25