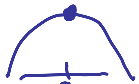
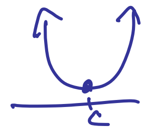
24.7 Applied Maximum + Minimum Problems



If $f'(c)=0$ and $f''(c)<0$ then maximum at $x=c$



If $f'(c)=0$ and $f''(c)>0$ then minimum.

p. 722

4 $P = 8x - .02x^2$

$P(x) = 8x - .02x^2$

Find critical values

$P'(x)=0$

$8 - .04x = 0$

$\frac{-.04x}{-.04} = \frac{-8}{-.04}$

$x = 200$ barrels

$P'(x) = 8 - .04x$

$P''(x) = -.04$

verify maximum at $x=200$

$P''(200) = -.04 < 0$ Maximum is at $x=200$.

Maximum Profit occurs when 200 barrels in a day.

Maximum Profit is $P(200) = 8(200) - .02(200)^2$ or \$800.

8

$h(t) = 16t^3 - 240t^2 + 10000$

$h'(t) = 48t^2 - 480t$

$h''(t) = 96t - 480$

Find critical values (set $h'(t)=0$)

$48t^2 - 480t = 0$

$48t(t-10) = 0$

$48t = 0 \qquad t - 10 = 0$

$t = 0 \qquad t = 10$

$h''(0) = 96(0) - 480 = -480 < 0$

so Maximum at 0.

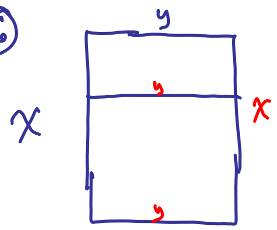
$h''(10) = 96(10) - 480 = 480 > 0$

so minimum at 10.

height when plane turns up is

$h(10) = 2000$ ft.

(16)



Amount of fence $= 240$ M

$2x + 3y = 240$

Solve for either variable.

$\frac{3y}{3} = \frac{240 - 2x}{3}$

$y = 80 - \frac{2}{3}x$

$\text{Area} = xy$ Maximize.

$A(x) = x\left(80 - \frac{2}{3}x\right)$

$= 80x - \frac{2}{3}x^2$

$A'(x) = 80 - \frac{4}{3}x$

$A''(x) = -\frac{4}{3}$

Find CV's. Set $A' = 0$

$80 - \frac{4}{3}x = 0$

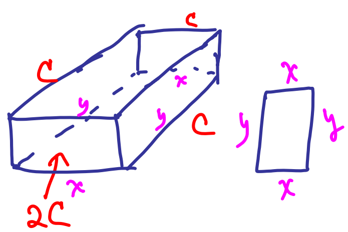
$\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right)x = (-80)\left(-\frac{3}{4}\right)$

$x = 60$

$y = 80 - \frac{2}{3}(60) = 40$

Dimensions are 60 m by 40 m.

(20)



$\text{Area} = 1350 \text{ m}^2$

$x \cdot y = 1350$

$y = \frac{1350}{x}$

Minimize cost $\left\{ \begin{array}{l} \text{Cost} = 2c \cdot x + c \cdot x + c \cdot y + c \cdot y \\ = 3cx + 2cy \end{array} \right.$

$C(x) = 3cx + 2c\left(\frac{1350}{x}\right)$

$= 3cx + 2700cx^{-1}$

$C'(x) = 3c - 2700cx^{-2}$

$C''(x) = 5400cx^{-3}$

Find CV's $C'(x) = 0$

$3c - 2700cx^{-2} = 0$

$3\cancel{c} = \frac{2700\cancel{c}}{x^2}$

$3x^2 = 2700$

$\sqrt{x^2} = \sqrt{900}$

$x = \cancel{\pm}30$

$C''(30) = 5400c(30)^{-3} > 0$ $\qquad \frac{5400}{30^3}c$

Minimum at $x = 30$

$y = \frac{1350}{30} = 45$

Dimensions are 30 m by 45 m.

p. 722–723: 3, 7, 13, 15, 25.

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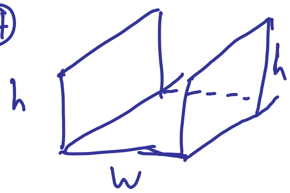


Cross Sectional Area.

$A = hw$

$2h + w = 36$

$w = 36 - 2h$

$A(h) = h(36-2h) = 36h - 2h^2$

$A'(h) = 36 - 4h$ $\qquad$ $A''(h) = -4$

Find CV's Set $A' = 0$

$36 - 4h = 0$

$-4h = -36$

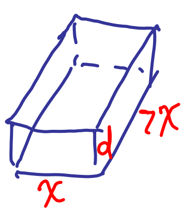
$h = 9$

$A''(9) = -4 < 0$ maximum at $h = 9$

$w = 36 - 2(9) = 18$

18 cm by 9 cm

26



Maximize Volume

Volume $= x \cdot 7x \cdot d = 7x^2 d$

$V(x) = 7x^{\cancel{2}}\left(\dfrac{980 - 7x^2}{16\cancel{x}}\right)$

$V(x) = \dfrac{1715}{4}x - \dfrac{49}{16}x^3$

$V'(x) = \dfrac{1715}{4} - \dfrac{49}{16}(3)x^2 = \dfrac{1715}{4} - \dfrac{147}{16}x^2$

$V''(x) = -\dfrac{147}{16}(2x) = -\dfrac{147}{8}x$

Find CV's Set $V' = 0$

$\dfrac{1715}{4} - \dfrac{147}{16}x^2 = 0$

$4\left(\dfrac{\cancel{16}}{147}\right)\dfrac{1715}{\cancel{4}} = \dfrac{147}{16}x^2\left(\dfrac{16}{147}\right)$

Area of sides & bottom is 980

$A =$ Bottom $+ 2$ Ends $+ 2$ sides

$980 = x(7x) + 2xd + 2(7x)d$

$980 = 7x^2 + 2xd + 14xd$

$980 = 7x^2 + 16xd$

$980 - 7x^2 = 16xd$

$d = \dfrac{980 - 7x^2}{16x}$

$x^2 = 46.\overline{6}$

$x = \cancel{\pm}\, 6.83$ ft

$V''(6.83) = -\frac{147}{8}(6.83) < 0$ so maximum at $x = 6.83$

$\text{Length} = 7x = 7(6.83) = 47.8$ ft

$\text{Depth} = d = \frac{980 - 7(6.83)^2}{16(6.83)} = 5.98$ ft

width 6.8 ft
Length 48 ft
depth 6.0 ft

p. 722-723: 3, 7, 13, 15, 25, 35