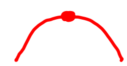
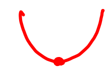
24.5 Using Derivatives in Curve Sketching

Relative maximum point (Local Maximum point)



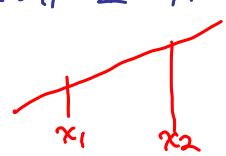
Relative maximum point will have a y value that is larger than all other y-values for a function on a given interval.

Relative Minimum point (Local Minimum Point)



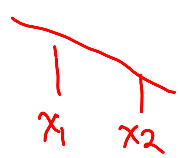
Relative minimum point will have a y-value that is smaller than all other y-values for a function on a given interval.

a function f is increasing on an interval I if $f(x_1) < f(x_2)$ where $x_1 < x_2$ on I.

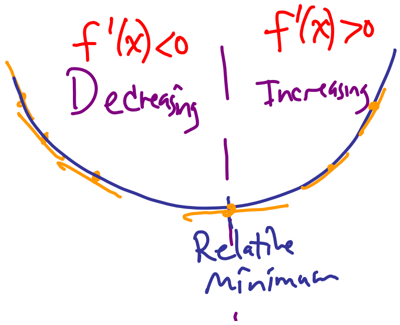


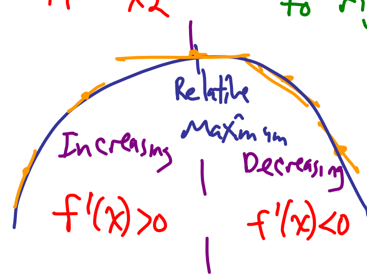
Graph goes up from left to right.

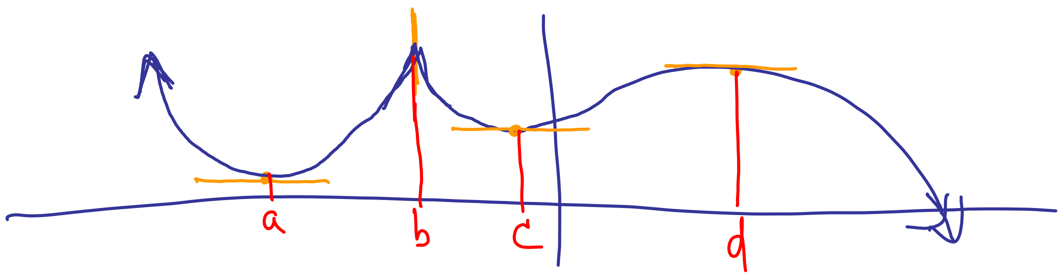
A function f is decreasing on an interval I if $f(x_1) > f(x_2)$ where $x_1 < x_2$ on I.



Graph goes down from left to right.







Relative minimums at $x=a$ & $x=c$

Relative maximums at $x=b$ & $x=d$

Intervals increasing (a,b) & (c,d)
$a<x<b$ $\quad c<x<d$

Intervals decreasing (b,c), (d,∞), & $(-\infty,a)$
$b<x<c$ $\quad x>d$ $\quad x<a$

Critical Point or Critical Value - Points where derivative is 0 or Does Not exist. Maximum or Minimums will occur at Critical points.

First Derivative Test ① If $f(x)$ is increasing ($f'(x)>0$) to the left of Critical value and decreasing ($f'(x)<0$) to the right then relative maximum will be at the critical value.

② If $f(x)$ is decreasing ($f'(x)<0$) to the left of Critical value and increasing ($f'(x)>0$) to the right then relative minimum will be at the critical value.

P. 712 ⑧ + ⑫ $y = x^4 - 6x^2 \Rightarrow y' = 4x^3 - 12x$

Find critical values $f'(x) = 0$

$4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$4x = 0 \qquad x^2 - 3 = 0$

$x = 0 \qquad x^2 = 3$

$x = \pm\sqrt{3}$

	-2 ($-\sqrt{3}$) -1	(0) 1	($\sqrt{3}$) 2	
$f'(x)$	$4(-2)^3 - 12(-2)$ $-8 < 0$ Neg	$4(-1)^3 - 12(-1)$ $+8 > 0$ Pos	$4(1)^3 - 12(1)$ $-8 < 0$ Neg	$4(2)^3 - 12(2)$ $8 > 0$ Pos
$f(x)$	Dec	Inc	Dec	Inc

⑧ $f(x)$ is increasing on $(-\sqrt{3}, 0) + (\sqrt{3}, \infty)$

$f(x)$ is decreasing on $(-\infty, -\sqrt{3}) + (0, \sqrt{3})$

⑫ Relative Minimums at $(-\sqrt{3}, f(-\sqrt{3}))$ and $(\sqrt{3}, f(\sqrt{3}))$ or $(-\sqrt{3}, -9)$ and $(\sqrt{3}, -9)$

Relative Maximum at $(0, f(0)) = (0, 0)$

$f(x) = x^4 - 6x^2$

$f(-\sqrt{3}) = f(\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 = 9 - 6(3) = -9$

If $f''(x) > 0$ on interval I then function $f(x)$ is Concave up on I. Looks like ∪.

If $f''(x) < 0$ on interval I then Function $f(x)$ is Concave Down on I.

Second Derivative Test If c is a critical value and

1) $f''(c) > 0$ then Relative minimum at $x = c$.
2) $f''(c) < 0$ then Relative Maximum at $x = c$
3) $f''(c) = 0$ then test Fails use First Derivative Test.

__Inflection Point__ is where function changes concavity
Will occur where $f''(x) = 0$ or $f''(x)$ is undefined.

16 & 20 $\quad y = x^4 - 6x^2 \Rightarrow y' = 4x^3 - 12x$

$y'' = 12x^2 - 12$

__Possible Inflection Points ($y'' = 0$)__

$12x^2 - 12 = 0$

$12(x^2 - 1) = 0$

$x = \pm 1$

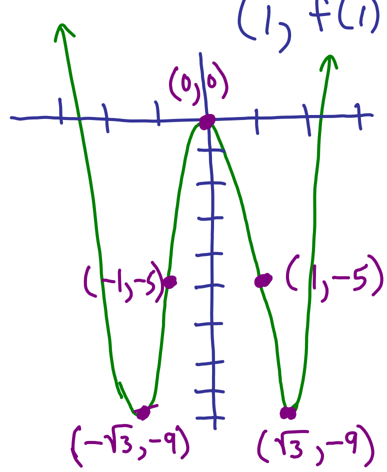
	-2	-1	0	1	2
$f''(x)$	$12(-2)^2 - 12 = 36 > 0$		$12(0)^2 - 12$ $-12 < 0$		$12(2)^2 - 12 = 36 > 0$
$f(x)$	Concave up		Concave Down		Concave up

Concave Up on $(-\infty, -1)$ & $(1, \infty)$

Concave Down on $(-1, 1)$

Inflection points: $(-1, f(-1)) = (-1, -5)$

$(1, f(1)) = (1, 5)$



p. 712-713: 5, 9, 13, 17, 23, 27

(24) $f(x) = y = x^3 - 9x^2 + 15x + 1 \Rightarrow f'(x) = 3x^2 - 18x + 15$

$f''(x) = 6x - 18$

Find Critical Values $(f'(x) = 0)$

$3x^2 - 18x + 15 = 0$

$3(x^2 - 6x + 5) = 0$

$3(x-5)(x-1) = 0$

$x = 1, 5$

	0	1	2	5	6
$f'(x)$	$3(0)^2 - 18(0) + 15 = 15 > 0$		$3(2)^2 - 18(2) + 15 = -9 < 0$		$3(6)^2 - 18(6) + 15 > 0$
$f(x)$	Increasing		Decreasing		Increasing

Increasing on $(-\infty, 1) + (5, \infty)$ Decreasing on $(1, 5)$

Relative maximum $(1, f(1)) = (1, 8)$

Relative minimum $(5, f(5)) = (5, -24)$

Find Possible Inflection Points $(f''(x) = 0)$

$6x - 18 = 0$

$x = 3$

	0	3	4
$f''(x)$	$6(0) - 18 = -18 < 0$		$6(4) - 18 = 6 > 0$
$f(x)$	Concave Down		Concave up

Concave up on $(3, \infty)$

Concave Down on $(-\infty, 3)$

Inflection Point $(3, f(3)) = (3, -8)$