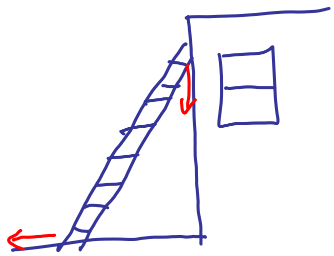
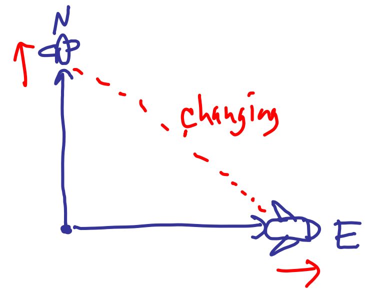
24.4 Related Rates

Derivatives gives the rate of change.





p. 705 ⑥

$$K = \frac{1}{2} m v^2$$

$m = 250\,kg \qquad V = 30.0\ m/s$

$v' = \frac{dv}{dt} = a = 5.00\ m/s^2$

$K' = \frac{dK}{dt} = ?$

$$\frac{d}{dt}(K) = \frac{d}{dt}\left(\frac{1}{2} m v^2\right)$$

$$\frac{dK}{dt} = \frac{1}{2} m\, 2v \frac{dv}{dt}$$

$$\frac{dK}{dt} = mv \frac{dv}{dt}$$

$$\frac{dK}{dt} = (250\,kg)\left(30.0\,\frac{m}{s}\right)\left(5.00\ m/s^2\right) = 37500\ J/s$$

(10)

$R_T = \frac{8R}{8+R}$ $\qquad \frac{dR}{dt} = R' = .30\ \Omega/\text{min}$ $\qquad R = 6.0\ \Omega$

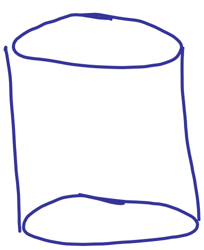
$\frac{d}{dt}(R_T) = \frac{d}{dt}\left(\frac{8R}{8+R}\right)$ $\qquad \frac{dR_T}{dt} = R_T' = ?$

$$\frac{dR_T}{dt} = \frac{(8+R)8\frac{dR}{dt} - (8R)\frac{dR}{dt}}{(8+R)^2} = \frac{64\frac{dR}{dt} + \cancel{8R\frac{dR}{dt}} - \cancel{8R\frac{dR}{dt}}}{(8+R)^2}$$

$$\frac{dR_T}{dt} = \frac{64R'}{(8+R)^2} \quad \text{or} \quad \frac{64\frac{dR}{dt}}{(8+R)^2}$$

$$\frac{dR_T}{dt} = \frac{64(.3)}{(8+6)^2} = .098\ \Omega/\text{min}$$

(16)



$V = \pi r^2 h$ $\qquad h = 15\text{ cm}$

$r' = .1\text{ mm/min} = .01\text{ cm/min}$

$r = \frac{9.5}{2} = 4.75\text{ cm}$

$V' = \pi h\, 2r\, r'$

$V' = \pi(15\text{ cm})\,2\,(4.75\text{ cm})\,(.01\text{ cm/min})$

$= 4.48\text{ cm}^3/\text{min}$

p. 705-706: 5, 7, 9, 15

p. 705-706: 5, 7. 9, 15