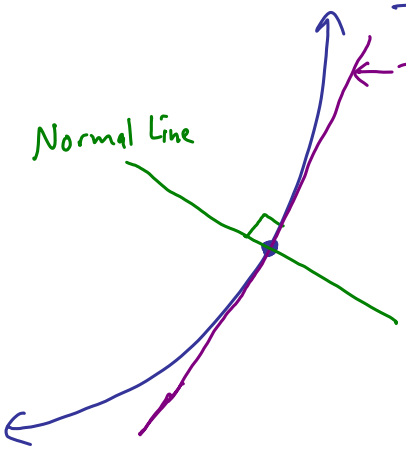
24.1 Tangents & Normals



$m_{tan} = y'$

$m_{normal} = -\frac{1}{y'}$ (Negative Reciprocal)

$y = 6x^2 - 2x$ Find equation of the tangent line & Normal line at $x = 2$.

$y' = 12x - 2$

$m_{tan} = y'(2) = 12(2) - 2 = 22$

$m_{norm} = -\frac{1}{22}$

$y(2) = 6(2)^2 - 2(2) = 20$

$(2, 20)$

$y - y_1 = m(x - x_1)$

$20\left(\frac{11}{11}\right) = \frac{220}{11}$

Tangent line

$y - 20 = 22(x - 2)$

$y - 20 = 22x - 44$

$y = 22x - 24$

Normal Line

$y - 20 = -\frac{1}{22}(x - 2)$

$y - 20 = -\frac{1}{22}x + \frac{1}{11}$

$y = -\frac{1}{22}x + \frac{221}{11}$

(12) $y = \sqrt{2x-9}$ tangent line with slope 1 $m_{tan} = 1$

$= (2x-9)^{1/2}$

$y' = \frac{1}{2}(2x-9)^{-1/2}(2) = \frac{1}{\sqrt{2x-9}}$

$m_{tan} = 1 \Rightarrow \sqrt{2x-9} \cdot 1 = \left(\frac{1}{\sqrt{2x-9}}\right)\sqrt{2x-9}$

$(\sqrt{2x-9})^2 = (1)^2$

$2x-9 = 1$

$2x = 10$

$x = 5$

$y = \sqrt{2(5)-9} = 1$

$(5,1)$ $m = 1$

$y - 1 = 1(x-5)$

$y = x - 5 + 1$

$y = x - 4$ tangent line

(18) $y^2 = 4x+4$ and $y^2 = 4-4x$

$2yy' = 4$ $2yy' = -4$

$y' = \frac{2}{y}$ $y' = -\frac{2}{y}$

at $(0,2)$ $m = \frac{2}{2} = 1$ $m = -\frac{2}{2} = -1$

$1 \cdot (-1) = -1$ so there are perpendicular

at $(0,-2)$ $m = \frac{2}{-2} = -1$ $m = -\frac{2}{-2} = 1$

so perpendicular.

$4x+4 = 4-4x$

$+4x \quad -4 \quad -4 \quad +4x$

$8x = 0$

$x = 0$

$y^2 = 4(0)+4$

$y^2 = 4$

$y = \pm 2$

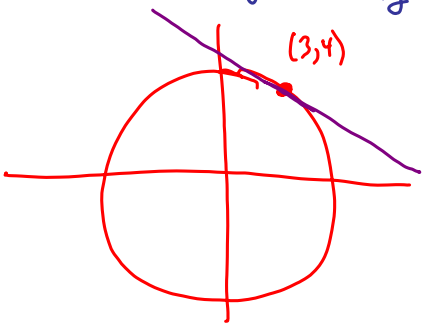
Intersection Points are $(0,2)$ & $(0,-2)$.

(22) $x^2+y^2=25$ at $(3,4)$

$2x+2yy'=0$

$2yy'=-2x$

$y'=-\frac{2x}{2y}=-\frac{x}{y}$



m_{tan} at $(3,4)$

$y'|_{(3,4)}=-\frac{3}{4}$

$y-4=-\frac{3}{4}(x-3)$

$y=-\frac{3}{4}x+\frac{9}{4}+4$

$y=-\frac{3}{4}x+\frac{25}{4}$

p.694-695: 3, 7, 9, 11, 13, 17, 21