23.8 Implicit Differentiation

$y = f(x) \Rightarrow$ Finding y' when function can be solved for y is called Explicit Differentiation.

$y = \sqrt{3x+2}$, $y = \frac{2x+1}{(3x-5x^2)^2}$, $x+2y=5$

$2y + y^2 = 3x$ cannot solve this for y to take derivative explicitly, so we will have to use implicit differentiation.

y is a function of x is $y(x)$

p.683 (4) $6x - 3y = 4$

$\frac{d}{dx}(6x) - \frac{d}{dx}(3y) = \frac{d}{dx}(4)$

$6 - 3\frac{dy}{dx} = 0$

solve for $\frac{dy}{dx}$: $\frac{-3\frac{dy}{dx}}{-3} = \frac{-6}{-3}$

y' or $\frac{dy}{dx} = 2$

$y' = \frac{dy}{dx}$

$6x - 3y = 4$

$\frac{-3y}{-3} = \frac{4-6x}{-3}$

$y = -\frac{4}{3} + 2x$

$y' = 2$

(8) $x^2 + 2y^2 - 11 = 0$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(2y^2) - \frac{d}{dx}(11) = \frac{d}{dx}(0)$

$2x + 2 \cdot 2y^{2-1} y' - 0 = 0$

$2x + 4y\,y' = 0$

$\frac{4y\,y'}{4y} = \frac{-2x}{4y}$

$y' = -\frac{x}{2y}$

$\frac{d}{dx}\left[(x^2+1)^2\right]$

$= 2(x^2+1)^1 (2x)$

$y \quad y'$

(10) $y^4 = 3x^3 - x$

$\frac{d}{dx}(y^4) = \frac{d}{dx}(3x^3 - x)$

$4y^3 \cdot y' = 9x^2 - 1$

$y' = \frac{9x^2 - 1}{4y^3}$

(12) $2y^3 - y = 7 - x^4$

$\frac{d}{dx}(2y^3 - y) = \frac{d}{dx}(7 - x^4)$

$6y^2 \cdot y' - y' = -4x^3$

$y'(6y^2 - 1) = -4x^3$

$y' = \frac{-4x^3}{6y^2 - 1}$

(14) $8y - xy - 7 = 0$

$\frac{d}{dx}(8y - \underbrace{xy}_{F\ S} - 7) = \frac{d}{dx}(0)$

$8y' - (xy' + y(1)) + 0 = 0$

$8y' - xy' - y = 0$

$8y' - xy' = y$

$y'(8 - x) = y$

$y' = \frac{y}{8 - x}$

(24) $2y + 5 - x^2 - y^3 = 0$ $\quad (2, -1)$

$2y' + 0 - 2x - 3y^2 y' = 0$

$2y' - 3y^2 y' = 2x$

$y'(2 - 3y^2) = 2x$

$y' = \frac{2x}{2 - 3y^2}$

$y'\Big|_{(2,-1)} = \frac{2(2)}{2 - 3(-1)^2} = \frac{4}{-1} = -4$

p683: 5, 7, 9, 11, 13, 15, 17, 25, 29, 31