23.7 Derivative of a Power of a Function

$$y = \frac{1}{x^2} = x^{-2}$$

$$\frac{dy}{dx} = -2x^{-2-1} = -2x^{-3}$$

$$= \boxed{\frac{-2}{x^3}}$$

$$y = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}}$$

$$\boxed{= \frac{1}{3}x^{-2/3} \text{ or } \frac{1}{3x^{2/3}} \text{ or } \frac{1}{3\sqrt[3]{x^2}}}$$

$$y = \frac{1}{x} + \sqrt{x} + \sqrt{2} = x^{-1} + x^{1/2} + \sqrt{2}$$

$$y' = -1x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} + 0 = -1x^{-2} + \frac{1}{2}x^{-1/2}$$

$$= -\frac{1}{x^2} + \frac{1}{2\sqrt{x}}$$

$$y = \frac{4}{x^3} + 6\sqrt[3]{x^4} = 4x^{-3} + 6x^{4/3}$$

$$y' = 4\left(-3x^{-3-1}\right) + \overset{2}{\cancel{6}}\left(\frac{4}{\cancel{3}}x^{\frac{4}{3}-1}\right) = -12x^{-4} + 8x^{1/3}$$

$$= \frac{-12}{x^4} + 8\sqrt[3]{x}$$

General Power Rule [Chain Rule]

$$f(x) = [u(x)]^n \qquad f'(x) = n[u(x)]^{n-1}u'(x)$$

$$y = (x^3+2)^4 \qquad y' = 4(x^3+2)^{4-1}(x^3+2)'$$

$$= 4(x^3+2)^3(3x^2)$$

$$= \boxed{12x^2(x^3+2)^3}$$

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(16) $y = 3(8x^2-1)^6$

$y' = 3 \cdot 6(8x^2-1)^5(8x^2-1)' = 18(8x^2-1)^5(16x) = \boxed{288x(8x^2-1)^5}$

(24) $y = \sqrt[3]{4x^6+2} = (4x^6+2)^{1/3}$

$y' = \frac{1}{3}(4x^6+2)^{1/3-1}(4x^6+2)' = \frac{1}{3}(4x^6+2)^{-2/3}(24x^5)$

$= 8x^5(4x^6+2)^{-2/3}$ or $\dfrac{8x^5}{(4x^6+2)^{2/3}}$

(20) $y = \dfrac{\pi^3}{\sqrt{1-3x}} = \pi^3(1-3x)^{-1/2}$

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(26) $y = \overset{F}{x^2}\,\overset{S}{(1-3x)^5}$ $(1-3x)'$

$y' = x^2\, 5 \cdot (1-3x)^4(-3) + (1-3x)^5(2x)$

$= -15x^2(1-3x)^4 + 2x(1-3x)^5$

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Do Not Simplify