23.6

Derivative of the Product of Two Functions

Product Rule

If $u(x) = F(x) \cdot S(x)$ (F(x): First, S(x): Second) then $\boxed{u'(x) = F(x)S'(x) + S(x)F'(x)}$

P.673 (6) $f(x) = (3x-2)(4x^2+3)$ (F(x) = $3x-2$, S(x) = $4x^2+3$)

$f'(x) = (3x-2)(8x) + (4x^2+3)(3)$ (F(x) $s'(x)$ + S(x) $F'(x)$)

$= 24x^2 - 16x + 12x^2 + 9$

$= 36x^2 - 16x + 9$

$f(x) = (3x-2)(4x^2+3) = 12x^3 + 9x - 8x^2 - 6$

(12) $y = (3x^2 - 4x + 1)(5 - 6x^2)$ (F(x) = $3x^2-4x+1$, S(x) = $5-6x^2$)

$y' = (3x^2-4x+1)(-12x) + (5-6x^2)(6x-4)$ (F(x) $s'(x)$ + S(x) $F'(x)$)

$= -36x^3 + 48x^2 - 12x + 30x - 20 - 36x^3 + 24x^2$

$= -72x^3 + 72x^2 + 18x - 20$

Derivative of the Quotient of Two Functions

Quotient Rule

If $u(x) = \dfrac{T(x)}{B(x)}$ (T(x): Top, B(x): Bottom)

$$\boxed{u'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{[B(x)]^2} \qquad u' = \frac{BT' - TB'}{B^2}}$$

(14) $y = \frac{4}{x^3}$ ← T, ↖ B

$y' = \frac{BT' - TB'}{B^2}$

$y' = \frac{x^3(4)' - 4(x^3)'}{(x^3)^2}$

$= \frac{x^3(0) - 4(3x^2)}{x^6} = \frac{-12x^3}{x^6} = \boxed{\frac{-12}{x^4}}$

(20) $y = \frac{2x^3}{4-x}$ ← T, ↖ B

$y' = \frac{(4-x)(2x^3)' - 2x^3(4-x)'}{(4-x)^2}$

$= \frac{(4-x)6x^2 - 2x^3(-1)}{(4-x)^2}$

$= \frac{24x^2 - 6x^3 + 2x^3}{(4-x)^2} = \boxed{\frac{24x^2 - 4x^3}{(4-x)^2}}$

(30) $y = \frac{2x^2 - 5x}{3x+2}$ ← T, ← B $\quad x = 2$

$y' = \frac{(3x+2)(4x-5) - (2x^2-5x)(3)}{(3x+2)^2} = \frac{12x^2 - 7x - 10 - 6x^2 + 15x}{(3x+2)^2}$

(B, T′, T, B′)

$= \frac{6x^2 + 8x - 10}{(3x+2)^2}$

$y'(2) = \frac{6(2)^2 + 8(2) - 10}{(3(2)+2)^2} = \frac{30}{64} = \boxed{\frac{15}{32}}$

p. 673-674: 3, 7, 9, 13, 15, 19, 25, 29, 39, 47