23.1 Limits

$$f(x)=\frac{x^3+2x^2-2x+3}{x+3}$$

$f(-3)$ is undefined.

x	$f(x)$
-3.5	16.75
-3.1	13.71
-3.01	13.07
-3.001	13.007
	$\downarrow$ 13

x	$f(x)$
-2.5	9.75
-2.9	12.31
-2.99	12.93
-2.999	12.993
	$\downarrow$ 13

$$\lim_{x\to -3}\frac{x^3+2x^2-2x+3}{x+3}=13$$

$$\boxed{\lim_{x\to a} f(x)=L}$$

limit as x approaches a of $f(x)$ is L.

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24 $\displaystyle\lim_{x\to 0}\frac{e^x-1}{x}=1$

x	$f(x)$
-1	$.6321$
$-.5$	$.7869$
$-.1$	$.9516$
$-.01$	$.9950$
$-.001$	$.9995$
	$\downarrow$ 1

x	$f(x)$
1	1.7183
$.5$	1.2974
$.1$	1.0517
$.01$	1.0050
$.001$	1.0005
	$\downarrow$ 1

If $f(a)$ exists then $\lim_{x \to a} f(x) = f(a)$

$$\lim_{x \to 2} 2x^2 - 3 = 2(2)^2 - 3 = 5$$

$$\lim_{x \to 0} 2x^2 - 3 = 2(0)^2 - 3 = -3$$

$$\lim_{x \to -1} 3x^3 - 2x + 1 = 3(-1)^3 - 2(-1) + 1 = -3 + 2 + 1 = 0$$

If $f(a) = \frac{0}{0}$, Algebraically rewrite function into a similar function that the value does exist for.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x \to 2} x + 2 = 2 + 2 = 4$$

$$\frac{2^2 - 4}{2 - 2} = \frac{0}{0}$$ ← Indeterminate Form.

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{3 - x} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{3-x}\,(-1)} = \lim_{x \to 3} -1(x+1) = -(3+1)$$

$$= -4$$

$$\frac{9 - 6 - 3}{3 - 3} = \frac{0}{0}$$

$$\lim_{x \to 1} (x-1)\sqrt{x^2 - 4}$$ Does Not Exist