Chapter 17 - Binomial Distribution

August 24, 2010

Suppose we pull out 10 M&M's from a large bag that contains 30% of a new "speckled" limited edition M&M's. What is the probability that 3 of the 10 are speckled?

This chapter deals with discrete random variables involving counts. We will restrict our discussion to a random variable that gives us the Binomial Probability Model (p. 436-440). The book also mentions Geometric and Poisson Probability Models.

Binomial Setting

1. There are a fixed number of trials, n.
2. The trials are independent.
3. There are only 2 possible outcomes (Success or Fail) in each trial.
4. The probability of success is the same for each trial, p.

Binomial Probability Model or Distribution: $B(n, p)$ or $\text{Binom}(n, p)$

n is the number of trials.
p is the probability of a success.
$q = 1 - p$ is the probability of a failure.
k is the number of successes.
$n - k$ is the number of failures.

Mean: $\mu = np$ Standard Deviation: $\sigma = \sqrt{npq}$

$P(X = k) = {}_nC_k \cdot p^k \cdot q^{n-k}$ where ${}_nC_k = \frac{n!}{k!(n-k)!}$. ${}_nC_k$ is read as "n choose k". You can access this function on your calculator by pushing the MATH button, then cursor over to the PRB menu option, and ${}_nC_k$ is the 3rd option. You will have to type $n\ {}_nC_k\ k$.

What about $P(X \le k)$?

Fortunately the Calculator has built in Binomial functions located in the DISTR menu (The same one we used for the Normalcdf function), by pushing the 2nd key then VARS (DISTR). The Binomial options are the 0: binompdf(and A:binomcdf(.

Binompdf(n,p,[k]) calculates $P(X = k) = {}_nC_k \cdot p^k \cdot q^{n-k}$ for a specific k.
The k is optional.if it is not included then the probability distribution for k going from 0 to n will be shown.

Binomcdf(n,p,k) Calculates $P(0) + P(1) + P(2) + \cdots + P(k)$ where each of the probabilities is calculated using the Binomial formula. This is a cumulative probability.

Go back to the M&M example from the beginning.

a) Find the Mean and Standard Deviation

b) Find the probability of pulling at exactly 3 speckled M&M's.

c) Find the probability of pulling out at most 3 Speckled M&M's.

d) Find the probability of pulling out at least 6 Speckled M&M's.

e) Find the probability of pulling out at least 3 Speckled M&M's.

f) Find the probability of pulling out more than 3 Speckled M&M's.

g) Find the probability of pulling out less than 3 Speckled M&M's.

For large n the Binomial Distribution will be approximately a Normal Distribution with $\mu = np$ and $\sigma = \sqrt{npq}$. How large an n? We want $np \geq 10$ and $nq \geq 10$. This means that there are at least 10 successes and 10 failures.