# Chapter 16 - Random Variables

August 24, 2010

There are many scenarios where probabilities are used to determine risk factors. Examples include Insurance, Casino, Lottery, Business, Medical, and other Sciences.

**Random Variable** is a variable whose value is a numerical outcome of a random phenomenon.

- Usually use capital letters at the end of the alphabet to denote a random variable like $X$ or $Y$.
- A particular value of a random variable will be denoted with a lower case letter.

**Two types of Random Variables:**

1. **Discrete random variable** has a finite number of distinct outcomes
   **Example:** Number of books this term.
2. **Continuous random variable** can take on any numerical value within a range of values.
   **Example:** Cost of books this term.

**Probability Model** consists of all values, $x$, of a random variable, $X$, along with the probabilities for each value denoted $P(X = x)$.

The **Expected Value** of a random variable is the value we expect a random variable to take on or the theoretical long-run average value. It is denoted $\mu$ for population mean or $E(x)$ for expected value. The expected value for a discrete random variable is found by summing each value by its probability.

$$\mu = E(x) = \sum x \cdot P(X = x)$$

**Note:** Every possible outcome most be included when finding $E(x)$ and the probability model must be legitimate.

**Examples:** p. 427: 2b

| *x* | 100 | 200 | 300 | 400 |
|---|---|---|---|---|
| *P(X=x)* | 0.1 | 0.2 | 0.5 | 0.2 |

p. 427-8: 8

The **Variance** of a random variable is the expected value of the squared deviation from the mean. The variance of a discrete random variable is found by calculating

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(X=x).$$

The **Standard Deviation** of a random variable is the square root of the variance.

$$\sigma = SD(X) = \sqrt{Var(X)}$$

**Example**: p. 428: 12b

Continuous random variables have means and variances, but this book does not show how to calculate them.

**More about Means and Variances:**

- Adding or subtracting a constant to each value of a random variable shifts the mean but does not change the variance or standard deviation:
  $E(X \pm c) = E(X) \pm c \qquad Var(X \pm c) = Var(X)$
- Multiplying each value of a random variable by a constant multiplies the mean by that constant and the variance by the square of the constant:
  $E(aX) = aE(X) \qquad Var(aX) = a^2 Var(X)$
- The mean of the sum of two random variables is the sum of the means:
  $E(X+Y) = E(X) + E(Y)$
- The mean of the difference of two random variables is the difference of the means:
  $E(X-Y) = E(X) - E(Y)$

- If two random variables are independent, then variance of their sum or differences is always the sum of the variances:

$$Var(X \pm Y) = Var(X) + Var(Y)$$

**Example:** p. 429: #28

| | Mean | SD |
|---|---|---|
| X | 80 | 12 |
| Y | 12 | 3 |