Chapter 14 - From Randomness To Probability

Random Phenomenon is a situation in which we know what outcomes can occur, but we do not know which outcome will occur. We cannot predict each outcome, but there will be a regular distribution over many repetitions.

For a random phenomenon each attempt or **trial** generates an outcome.

Sample Space is the set of all possible outcomes of a random phenomenon.

Rolling Two coins:

Number of heads on 2 coins:

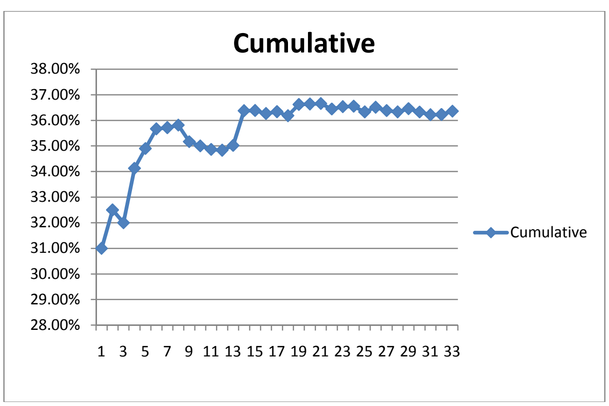
Number when rolling 2 dice:

Event - A set of outcomes of a random phenomenon.

Rolling an odd on 2 dice: Rolling at most 5 on 2 dice:

Individual trials are **independent** if the outcome of one trial does not influence or change the outcome of another.

The Law of Large Numbers says that the long-run relative frequency of repeated independent events gets closer and closer to the true relative frequency as the number of trials increases.



This does not mean that a random phenomenon is supposed to compensate for the past. If we had 6 numbers in a row that were less than or equal to 7 the next throw does NOT have a better chance of landing with a number greater than 7. What is the chance that the next throw has a number less than or equal to 7?

Probability of an outcome is the proportion of times the outcome occurs over a long series of repetitions.

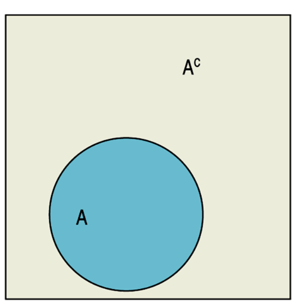
Probability Rules:

1. A probability is a number between and including 0 and 1.
   For any event A, $0 \le P(A) \le 1$. The probability of an event that will never occur is 0, while the probability of an event that will always occur is 1.

2. The probability of all outcomes together is 1. $P(S) = 1$

3. The **Complement of A**, denoted A^c, is the set of all outcomes that are not in the event A.

   Complement Rule: The probability that an event occurs is 1 minus the probability the event does not occur.



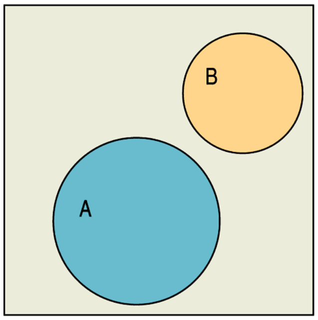
The set **A** and its complement.

$$P(A) = 1 - P(A^c)$$

or $P(A^c) = 1 - P(A)$

or $P(A) + P(A^c) = 1$

4. Two events that have nothing in common are called **Disjoint** or **Mutually Exclusive**.



Two disjoint sets, **A** and **B.**

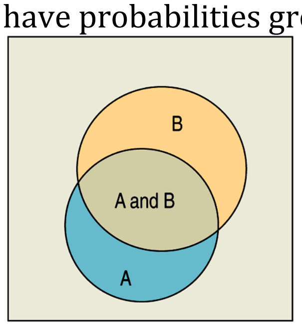
Addition Rule: For two disjoint events A and B, the probability that one or the other occurs is the sum of the probabilities of the two events.

$$P(A \text{ or } B) = P(A) + P(B)$$

5. **Multiplication Rule**: For two independent events A and B, the probability that both A and B occur is the product of the probabilities of the two events.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Two independent events A and B are not disjoint, provided the two events have probabilities greater than 0.



Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

What Can Go Wrong?

- Beware of probabilities that do not add up to 1.
- Do not add probabilities of events that are not disjoint.
- Do not multiply probabilities of events if they are not independent.
- Do not confuse disjoint and independent.