Chapter 17 - Testing Hypotheses About Proportions

In Statistics, a **hypothesis** proposes a model for the world we are interested in. This will be some statement about the parameters in a population.

Significance Test (or Hypothesis Testing) is a formal way to compare observed data (sample) with a hypothesis whose truth we want to assess.

1. If the data is consistent with the model, then we have no reason to disbelieve the hypothesis. The data supports the hypothesis, but does not prove the hypothesis.
2. If the data is inconsistent with the model, then we need to decide if the data is inconsistent enough to disbelieve the hypothesis. If inconsistent enough the data causes us to reject the hypothesis.

There are Four basic parts to Hypothesis Testing

1. State the Hypothesis
2. Determine the Model
3. Perform the Mechanics
   - Calculate the Test Statistic
   - Find P-Value
4. State Conclusion

17.3 The Reasoning of Hypothesis Testing

1. State the Hypothesis (17.1)

Determine the Hypothesis or Original Claim that is being tested. Write in symbolic form along with the opposite claim.

Null Hypothesis (H_0) is the statement of no effect or no difference.
Usually something of the form:
H_0: parameter = hypothesized value.

Alternative Hypothesis (H_A) is the hypothesis we accept if the null is rejected.
Usually something in one of the following forms:
H_A: parameter > hypothesized value (**Right-Tailed Alternative**),
H_A: parameter < hypothesized value (**Left-Tailed Alternative**), or
H_A: parameter ≠ hypothesized value (**Two-Tailed Alternative**).

2. Determine the Model

Each Model has some name that should be reported. Also, any conditions for a test must be satisfied.
The test for proportions is the **one-proportion *z*-test**.

3. **Perform the Mechanics**

Usually this will be done by computer software or in our case the calculator, but we will learn the calculations involved to better understand the process.

a) **Calculate the Test Statistic**

The formula for the test statistic will vary depending on what test is being used. The formula for the one-proportion *z*-test is presented here.

To test the Null Hypothesis: $H_O : p = p_o$

We use the test statistic (t.s.): $z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}}$

Why do we use $SD_{\hat{p}} = \sqrt{\dfrac{p_O q_O}{n}}$?

b) **Find the P-Value**

The **P-Value** is the probability that the test statistic will take a value at least as extreme as the observed value given that H_0 is true. **(17.2)**

The smaller the p-value the stronger the evidence is against H_0. (The smaller the p-value the less likely it will occur.)

Right-Tail: $\text{p-value} = P(z > \text{test statistic})$

Left-Tail: $\text{p-value} = P(z < \text{t.s.})$

Two-Tail: $\text{p-value} = 2 \cdot P(z > \text{t.s.})$ or $2 \cdot P(z < \text{t.s.})$ **(17.4)**

4. **State the Conclusion** **(17.5)**

The conclusion is a statement about the hypothesis.
The conclusion must state either that we reject the null hypothesis or fail to reject the null hypothesis.
The conclusion should be stated in context of the original problem.

Significance Level (α) - The fixed value, determined in advance, that will be used to decide if the p-value is too extreme or not. (If not given use $\alpha = 0.05$) $[\alpha = 1 - C]$

- If $\text{p-value} \leq \alpha$ then data is statistically significant at the α level and we reject the H_0 and accept the H_A.

- If $\text{p-value} > \alpha$ then data does not provide sufficient evidence to reject the null hypothesis. We fail to reject the H_0. This does not prove H_0 is true, but data supports H_0.

Statistically Significant does not mean importance. In statistics significant is used as in identifying (signifying) a level or standard.

If $\text{p-value} = 0.03$, this is significant at $\alpha = 0.05$ but not at $\alpha = 0.01$.