There are many scenarios where probabilities are used to determine risk factors. Examples include Insurance, Casino, Lottery, Business, Medical, and other Sciences.

14.1 Expected Value: Center

Random Variable is a variable whose value is a numerical outcome of a random phenomenon.

- Usually use capital letters at the end of the alphabet to denote a random variable like X or Y.
- A particular value of a random variable will be denoted with a lower case letter.

Two types of Random Variables:

1. **Discrete random variable** has a finite number of distinct outcomes
   Example: Number of books this term.

2. **Continuous random variable** can take on any numerical value within a range of values.
   Example: Cost of books this term.

Probability Model consists of all values, x, of a random variable, X, along with the probabilities for each value denoted $P(X = x)$.

The **Expected Value** of a random variable is the value we expect a random variable to take on or the theoretical long-run average value. It is denoted μ for population mean or $E(x)$ for expected value. The expected value for a discrete random variable is found by summing each value by its probability.

$$\mu = E(x) = \sum x \cdot P(X = x)$$

Note: Every possible outcome most be included when finding $E(x)$ and the probability model must be legitimate.

Examples: p. 388: 16b

x	100	200	300	400
$P(X=x)$	0.1	0.2	0.5	0.2

x	$P(x)$	$x \cdot P(x)$
100	.1	100x.1 = 10
200	.2	200x.2 = 40
300	.5	300x.5 = 150
400	.2	400x.2 = 80
		280

$\mu = 280$

or

$E(x) = 280$

p. 388: 20

\$100 cash prize \$5 per throw

Person is willing to spend \$20 or 4 throws.

$P(\text{Hit}) = .1$ $P(\text{Miss}) = 1 - .1 = .9$

a)

Number of Darts	1	2	3	4
P(Number of Darts)	.1	.09	.081	.729

Miss Hit Miss Miss Hit $1-(.1+.09+.081)$

.9x.1 .9x.9x.1

b)

x	$P(x)$	$x \cdot P(x)$
1	.1	1x.1 = .1
2	.09	2x.09 = .18
3	.081	3x.081 = .243
4	.729	4x.729 = 2.916
		3.439

$\mu = 3.439$

On average a person will throw 3.439 darts if using this strategy.

c)

	Amount Won	P(Amount Won)	$x \cdot P(x)$
Won 1st	\$95	.1	$95 \times .1 = 9.5$
Won 2nd	\$90	.09	$90 \times .09 = 8.1$
Won 3rd	\$85	.081	$85 \times .081 = 6.885$
Won 4th	\$80	.0729	$80 \times .0729 = 5.832$
Didn't win	-\$20	.6561	$-20 \times .6561 = -13.122$

17.195 ← μ or $E(X)$

Won 4th $.9 \times .9 \times .9 \times .1 = .0729$ $\quad .729$

Didn't win $.9 \times .9 \times .9 \times .9 = .9^4 = .6561$

ON Average a player is expected to win \$17.20 if this strategy is used.

14.2 Standard Deviation

The **Variance** of a random variable is the expected value of the squared deviation from the mean. The variance of a discrete random variable is found by calculating

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \cdot P(x).$$

The **Standard Deviation** of a random variable is the square root of the variance.

$$\sigma = SD(X) = \sqrt{Var(X)}$$

ON Calculator p.387
1-Var Stat L_1, L_2

Example: p. 388: 24b $\quad \mu = E(x) = 280$

x (L1)	P(x) (L2)	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
100	.1	$(100-280)^2 = (-180)^2 = 32400$	$32400 \times .1 = 3240$
200	.2	$(200-280)^2 = (-80)^2 = 6400$	$6400 \times .2 = 1280$
300	.5	$(300-280)^2 = 20^2 = 400$	$400 \times .5 = 200$
400	.2	$(400-280)^2 = 120^2 = 14400$	$14400 \times .2 = 2880$
			7600

$\sigma^2 = Var(x) = 7600$

$\sigma = SD(x) = \sqrt{7600} = 87.18$

14.3 Combining Random Variables

Continuous random variables have means and variances, but this book does not show how to calculate them.

More about Means and Variances:

- Adding or subtracting a constant to each value of a random variable shifts the mean but does not change the variance or standard deviation:

$$E(X \pm c) = E(X) \pm c \quad Var(X \pm c) = Var(X) \quad SD(X \pm c) = SD(X)$$

- Multiplying each value of a random variable by a constant multiplies the mean by that constant and the variance by the square of the constant:

$$E(aX) = aE(X) \qquad Var(aX) = a^2 Var(X) \quad SD(aX) = |a| SD(X)$$

- The mean of the sum of two random variables is the sum of the means:
- The mean of the difference of two random variables is the difference of the means:
- If two random variables are independent, then variance of their sum or differences is always the sum of the variances:

$$E(X \pm Y) = E(X) \pm E(Y) \qquad Var(X \pm Y) = Var(X) + Var(Y)$$

$$SD(X \pm Y) = \sqrt{Var(X \pm Y)} = \sqrt{Var(X) + Var(Y)}$$

Example: p. 390: #40

$\mu = E(X) \quad \sigma = SD(X) \quad \sigma^2 = Var(X)$

	Mean	SD	Var
X	80	12	144
Y	12	3	9

a) $X - 20$ $\quad E(X-20) = E(X) - 20 = 80 - 20 = 60$

$SD(X-20) = SD(X) = 12$

b) $.5Y$ $\quad E(.5Y) = .5E(Y) = .5 \times 12 = 6$

$SD(.5Y) = |.5| SD(Y) = .5 \times 3 = 1.5$

d) $X - Y$ $\quad E(X-Y) = E(X) - E(Y) = 80 - 12 = 68$

$SD(X-Y) = \sqrt{Var(X-Y)} = \sqrt{Var X + Var Y} = \sqrt{144+9} = \sqrt{153} = 12.37$

e) $Y_1 + Y_2$ $\quad E(Y_1+Y_2) = E(Y) + E(Y) = 12 + 12 = 24$

$SD(Y_1+Y_2) = \sqrt{Var(Y_1+Y_2)} = \sqrt{Var Y + Var Y} = \sqrt{9+9} = \sqrt{18} = 4.24$

14.4 The Binomial Model

Suppose we pull out 10 M&M's from a large bag that contains 30% of a new "speckled" limited edition M&M's. What is the probability that 3 of the 10 are speckled?

This chapter deals with discrete random variables involving counts. We will restrict our discussion to a random variable that gives us the Binomial Probability Model (p. 436-440). The book also mentions Geometric and Poisson Probability Models.

Binomial Setting

1. There are a fixed number of trials, n.
2. The trials are independent.
3. There are only 2 possible outcomes (Success or Fail) in each trial.
4. The probability of success is the same for each trial, p.

Binomial Probability Model or Distribution: $B(n,p)$ or $\text{Binom}(n,p)$

n is the number of trials.
p is the probability of a success.
$q = 1 - p$ is the probability of a failure.
k is the number of successes.
$n-k$ is the number of failures.

or Expected Value
Mean: $\mu = np$ Standard Deviation: $\sigma = \sqrt{npq}$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$P(X=k) = {}_nC_k \cdot p^k \cdot q^{n-k}$ where ${}_nC_k = \dfrac{n!}{k!(n-k)!}$. ${}_nC_k$ is read as "n choose k". You can access this function on your calculator by pushing the MATH button, then cursor over to the PRB menu option, and ${}_nC_k$ is the 3rd option. You will have to type $n \; {}_nC_k \; k$.

$P(X=3) = {}_{10}C_3 \, .3^3 \, .7^7 = 120(.3)^3(.7)^7 = .2668$

What about $P\left(X \le k\right)$?

Fortunately the Calculator has built in Binomial functions located in the DISTR menu (The same one we used for the Normalcdf function), by pushing the 2nd key then VARS (DISTR). The Binomial options are the 0: binompdf(and A:binomcdf(.

Binompdf(n,p,[k]) calculates $P\left(X = k\right) = {}_nC_k \cdot p^k \cdot q^{n-k}$ for a specific k.

The k is optional.if it is not included then the probability distribution for k going from 0 to n will be shown. $P(X \le k)$

Binomcdf(n,p,k) Calculates $P\left(0\right)+P\left(1\right)+P\left(2\right)+\cdots+P\left(k\right)$ where each of the probabilities is calculated using the Binomial formula. This is a cumulative probability.

Go back to the M&M example from the beginning. $n=10$ $p=.3$ $q=.7$

a) Find the Mean and Standard Deviation

$\mu = np = 10 * .3 = 3$

$\sigma = \sqrt{npq} = \sqrt{10 \times .3 \times .7} = 1.45$

b) Find the probability of pulling out exactly 3 speckled M&M's.

Binompdf (10, .3, 3)

$P(X=3) = .2668$

c) Find the probability of pulling out at most 3 Speckled M&M's.

$P(X \le 3) =$ Binomcdf (10, .3, 3)

$= .6496$

$P(0)+P(1)+P(2)+P(3)$

6, 7, 8, 9, 10

d) Find the probability of pulling out at least 6 Speckled M&M's.

$P(X \ge 6) = 1 - P(X \le 5)$

$= 1 -$ Binomcdf (10, .3, 5)

$= .0473$

3, 4, 5, ...

e) Find the probability of pulling out at least 3 Speckled M&M's.

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 1 - \text{Binomcdf}(10, .3, 2)$$
$$= .6172$$

4, 5, 6, ...

f) Find the probability of pulling out more than 3 Speckled M&M's.

$$P(X > 3) = 1 - P(X \leq 3)$$
$$= 1 - \text{Binomcdf}(10, .3, 3)$$
$$= .3504$$

0, 1, 2

g) Find the probability of pulling out less than 3 Speckled M&M's.

$$P(X < 3) = P(X \leq 2)$$
$$= \text{Binomcdf}(10, .3, 2)$$
$$= .3828$$

$\mu = np \quad \sigma = \sqrt{npq}$

$P(X = k) = \text{Binompdf}(n, p, k)$

$P(X \neq k) = 1 - \text{Binompdf}(n, p, k)$

$P(X \leq k) = \text{Binomcdf}(n, p, k)$

$P(X < k) = \text{Binomcdf}(n, p, k-1)$

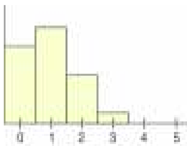
$P(X > k) = 1 - \text{Binomcdf}(n, p, k)$

$P(X \geq k) = 1 - \text{Binomcdf}(n, p, k-1)$

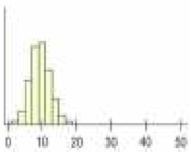
14.5 Modeling the Binomial with a Normal Model

For large n the Binomial Distribution will be approximately a Normal Distribution with $\mu = np$ and $\sigma = \sqrt{npq}$. How large an n? We want $np \geq 10$ and $nq \geq 10$. This means that there are at least 10 successes and 10 failures.

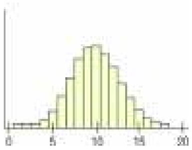
p.376



$n = 5$ $\quad$ $n \cdot p = 5(.2) = 1$



$n = 20$ $\quad$ $np = 20(.2) = 4$



$n = 50$ $\quad$ $np = 50(.2) = 10$

14.7 Continuous Random Variables

The calculation of Expected Value and Standard Deviation for a Continuous Random Variable can be done but will not be shown in this book.

p.388.

8. Soccer: A soccer team estimates that they will score on 8% of the corner kicks. In next week's game, the team hopes to kick 15 corner kicks. What are the chances that they will score on 2 of those opportunities?

$n = 15 \quad p = .08 \quad q = .92$

$\mu = np = 15 \times .08 = 1.2$

$\sigma = \sqrt{npq} = \sqrt{15 \times .08 \times .92} = 1.05$

$P(X = 2) = \text{Binompdf}(15, .08, 2) = .2273$

10. Soccer again: If this team has 200 corner kicks over the season, what are the chances that they score more than 22 times?

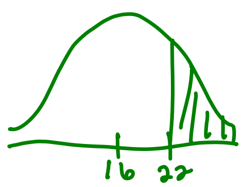
$n = 200 \quad p = .08 \quad q = .92$

$\mu = np = 200 \times .08 = 16 \qquad \sigma = \sqrt{npq} = \sqrt{200 \times .08 \times .92} = 3.837$

$P(X > 22) = 1 - P(X \le 22)$

23, 24, 25, ... $= 1 - \text{Binomcdf}(200, .08, 22) = .0507$

Normal Approximation



$P(X > 22) \approx \text{Normalcdf}(22, 100000, 16, 3.837)$

$\approx .0589$