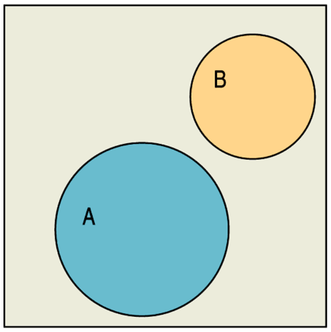
Chapter 13 - Probability Rules!

13.1 The General Addition Rule

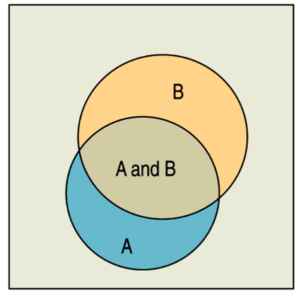
Addition Rule: For two disjoint events A and B,

$$P(A \text{ or } B) = P(A) + P(B)$$

What happens if A and B are not disjoint?



Two disjoint sets, **A** and **B.**



Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

Notice the area that includes A and B is included twice.

General Addition Rule: For any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Examples:

1. Using a standard deck of 52 cards (2 colors {Black and Red}, 4 suits {Clubs, Spades, Diamonds, and Hearts}, 13 types {Ace-10, Jack, Queen, King}, draw a card at random and find the following probabilities.

- The card is a 10 or a 3. Disjoint Events

$$P(10 \text{ or } 3) = P(10) + P(3) - P(10 \text{ and } 3)$$
$$= \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \frac{2}{13} \approx .1538$$

- The card is a 10 or a Spade. Not Disjoint Events

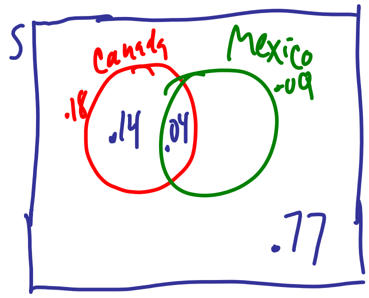
P(Both)

$$P(10 \text{ or Spade}) = P(10) + P(\text{Spade}) - P(10 \text{ and spade})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} \approx .3077$$

(10 of spades) is in both so subtract one

$P(\text{Canada}) = .18 \quad P(\text{Mexico}) = .09 \quad P(\text{Both}) = .04$

2. p. 357, #16



a) $P(\text{Canada but not Mexico}) = .18 - .04 = .14$

b) $P(\text{Canada or Mexico}) = P(\text{Canada}) + P(\text{Mexico}) - P(\text{Both})$
$= .18 + .09 - .04 = .23$

c) $P(\text{Neither Canada Nor Mexico}) = 1 - .23 = .77$

3. p. 357, #20

		Birth Order		
		1 or only	2 or more	Total
College	Arts & Sciences	34	23	57
	Agriculture	52	41	93
	Human Ecology	15	28	43
	Other	12	18	30
	Total	113	110	223

a) $P(\text{Human Ecology}) = \frac{43}{223}$

b) $P(\text{1st Born}) = \frac{113}{223}$

c) $P(\text{1st Born AND Human Ecology}) = \frac{15}{223}$

d) $P(\text{1st Born } \underline{\text{OR}} \text{ Human Ecology}) = P(\text{1st Born}) + P(\text{H.E.}) - P(\text{1st Born AND H.E.})$

$= \frac{113}{223} + \frac{43}{223} - \frac{15}{223}$

$= \frac{141}{223} \approx .632$

13.2 Conditional Probability and the General Multiplication Rule

Conditional Probability gives the probability of one event under the condition another event has occurred.

$$P(B \overset{\text{"Given"}}{|} A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{or} \quad \frac{\text{Number of } A \text{ and } B}{\text{Number of } A}$$

Probability that event B occurs given that event A has already occurred.

Examples:

1. p. 357, #22

	Cats	Dogs	
male	6	8	14
female	12	16	28
	18	24	42

a) $P(\text{Male} \mid \text{Cat}) = \dfrac{P(\text{Male and Cat})}{P(\text{Cat})} = \dfrac{6/\cancel{42}}{18/\cancel{42}} = \dfrac{6}{18} = \dfrac{1}{3} \approx .333$

$$\frac{6}{42} \div \frac{18}{42} = \frac{6}{\cancel{42}} \times \frac{\cancel{42}}{18} = \frac{6}{18} = \frac{1}{3}$$

b) $P(\text{Cat} \mid \text{female}) = \dfrac{P(\text{female and cat})}{P(\text{female})} = \dfrac{12/\cancel{42}}{28/\cancel{42}} = \dfrac{12}{28} = \dfrac{3}{7} \approx .429$

c) $P(\text{female} \mid \text{Dog}) = \dfrac{P(\text{female and Dog})}{P(\text{Dog})} = \dfrac{16/\cancel{42}}{24/\cancel{42}} = \dfrac{16}{24} = \dfrac{2}{3} \approx .667$

2. p. 357, #26

College		Birth Order: 1 or only	2 or more	Total
	Arts & Sciences	34	23	57
	Agriculture	52	41	93
	Human Ecology	15	28	43
	Other	12	18	30
	Total	113	110	223

a) $P(\text{Arts + Sciences AND 2nd or More}) = \dfrac{23}{223} \approx .103$

b) $P(\text{2nd or More} \mid \text{Arts + Sciences}) = \dfrac{23}{57} \approx .404$

c) $P(\text{Arts + Sciences} \mid \text{2nd or More}) = \dfrac{23}{110}$

d) $P(\text{Agriculture} \mid \text{1st or Only}) = \dfrac{52}{113}$

e) $P(\text{1st or only} \mid \text{Agriculture}) = \dfrac{52}{93}$

Multiplication Rule: For two independent events A and B,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

What happens if A and B are not independent?

General Multiplication Rule: For any two events A and B,

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Or

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

13.3 Independence

Events A and B are **independent** if the probability of one does not change if the other event has already occurred – i.e., $P(B \mid A) = P(B)$.

Example:
p. 359, #36

$P(\text{Canada}) = .18 \quad P(\text{Mexico}) = .09 \quad P(\text{Both}) = .04$

a) $P(\text{Canada} \mid \text{Mexico}) = \dfrac{P(\text{Both})}{P(\text{mexico})} = \dfrac{.04}{.09} = .444$

b) Not Disjoint since $P(\text{Both}) = .04$

c) $P(\text{Canada} \mid \text{Mexico}) = .444 \qquad P(\text{canada}) = .18$

Not independent since $P(\text{Canada} \mid \text{Mexico}) \neq P(\text{canada})$

Independent ≠ Disjoint

13.4 Picturing Probability

The General Multiplication Rule can be seen in **Drawing Without Replacement**.
This just means that once an object is drawn it is not put back into the pool. This is not a problem for large populations, but it is important when working with small populations.

15 Marbles

Example: A jar contains 6 Blue Marbles, 4 Red Marbles, and 5 Green Marbles. What is the probability of randomly pulling out 2 Blue Marbles?

- With Replacement: Independent Events

1st Blue, 2nd Blue

$$P(2 \text{ Blue Marbles}) = \frac{6}{15} \times \frac{6}{15} = \frac{36}{225} = \frac{4}{25} = .16$$

- Without Replacement: Not Independent

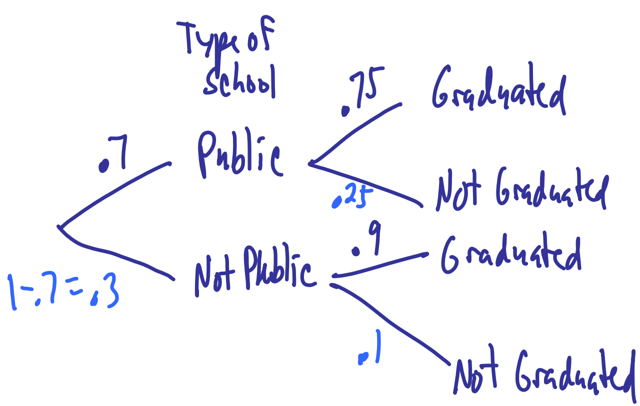
1st Blue, 2nd Blue

$$P(2 \text{ Blue Marbles}) = \frac{6}{15} \times \frac{5}{14} = \frac{30}{210} = \frac{1}{7} \approx .143$$

$$P\left(\begin{matrix}\text{1st}\\ \text{Blue}\end{matrix}\right) \times P\left(\begin{matrix}\text{2nd}\\ \text{Blue}\end{matrix} \middle| \begin{matrix}\text{1st}\\ \text{Blue}\end{matrix}\right)$$

Tree Diagrams can also help in working with conditional probabilities. The branches of the trees are the different events given that other events have occurred.

Example: p. 360: 48 & 50



$.7 \times .75 = .525$ *

$.7 \times .25 = .175$

$.3 \times .9 = .27$ *

$.3 \times .1 = .03$

a) $P(\text{Graduated} \mid \text{Public}) = .75$ $\quad P(\text{Graduated} \mid \text{Not Public}) = .9$

Not Independent since graduation rates from those that attended Public School is different than those that did not attend Public Schools. If Independent rates would be equal.

b) $P(\text{Graduated}) = .525 + .27 = .795$ $\quad$ 79.5% of freshmen graduated.

(50) $P(\text{Public} \mid \text{Graduated}) = \dfrac{P(\text{Public} \,\&\, \text{Graduated})}{P(\text{Graduated})} = \dfrac{.525}{.795} \approx .660$

66.0% of students who graduated went to a Public School.