# Chapter 12 - From Randomness To Probability



**12.1 Random Phenomenon** is a situation in which we know what outcomes can occur, but we do not know which outcome will occur. We cannot predict each outcome, but there will be a regular distribution over many repetitions.

For a random phenomenon each attempt or **trial** generates an outcome.

**Sample Space** is the set of all possible outcomes of a random phenomenon.

Rolling Two coins: $S = \{TT, TH, HT, HH\}$

Number of heads on 2 coins: $S = \{0, 1, 2\}$

Number when rolling 2 dice: $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

**Event** - A set of outcomes of a random phenomenon.

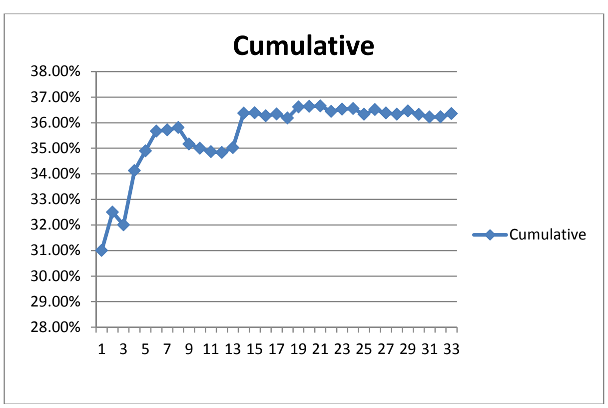
(at least 10 $\geq 10$)

Rolling an odd on 2 dice: $A = \{3, 5, 7, 9, 11\}$

Rolling at most 5 ($\leq 5$) on 2 dice: $B = \{2, 3, 4, 5\}$

Individual trials are **independent** if the outcome of one trial does not influence or change the outcome of another.

**The Law of Large Numbers** says that the long-run relative frequency of repeated independent events gets closer and closer to the true relative frequency as the number of trials increases.




This does not mean that a random phenomenon is supposed to compensate for the past. If we had 6 numbers in a row that were less than or equal to 7 the next throw does NOT have a better chance of landing with a number greater than 7. What is the chance that the next throw has a number less than or equal to 7?

## 12.2 Modeling Probability

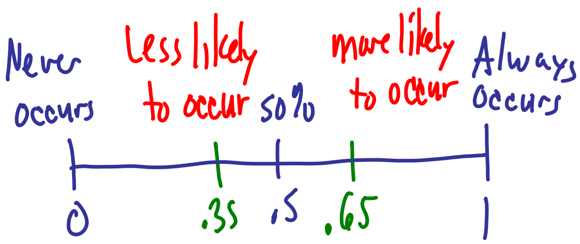
**Probability** of an outcome is the proportion of times the outcome occurs over a long series of repetitions.

$$P(A) = \frac{\#\text{ outcomes in } A}{\#\text{ of possible outcomes}}$$

## 12.3 Formal Probability

**Probability Rules:**

1. A probability is a number between and including 0 and 1.
   For any event $A$, $0 \le P(A) \le 1$. The probability of an event that will never occur is 0, while the probability of an event that will always occur is 1.

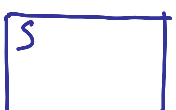



Rolling 2 6-sided Die

$P(13) = 0$

$P(\text{at least } 2) = 1$

2. The probability of all outcomes together is 1. $P(S) = 1$

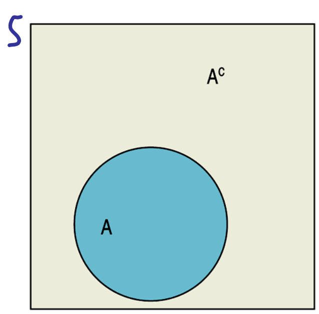



Area of S is 1

3. The **Complement of A**, denoted $A^c$, is the set of all outcomes that are not in the event $A$.

$P(A^c) = P(\text{not } A) = P(\bar{A})$

**Complement Rule**: The probability that an event occurs is 1 minus the probability the event does not occur.




The set **A** and its complement.

$$P(A) = 1 - P(A^c)$$

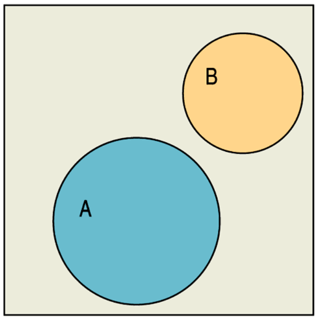
$$\text{or } P(A^c) = 1 - P(A)$$

$$\text{or } P(A) + P(A^c) = 1$$

$P(\text{number} \le 7) = .35$

$P(\text{Number} > 7) = 1 - .35 = .65$

4. Two events that have nothing in common are called **Disjoint** or **Mutually Exclusive**.




Two disjoint sets, **A** and **B.**

Circles do not overlap

Tossing 2 Dice

$A = \{3, 5, 7, 9, 11\}$ $B = \{2, 3, 4, 5\}$

A & B are not disjoint since both have 3 & 5.

$C = \{2, 4, 6, 8, 10, 12\}$ A & C are disjoint or mutually exclusive

**Addition Rule**: For two disjoint events $A$ and $B$, the probability that one or the other occurs is the sum of the probabilities of the two events.

$$P(A \text{ or } B) = P(A) + P(B)$$

Generalize for more than 2 events.

Jar with 3 red marbles, 5 green marbles, & 2 orange marbles.

$$P(\text{Red or Green}) = P(\text{Red}) + P(\text{Green})$$

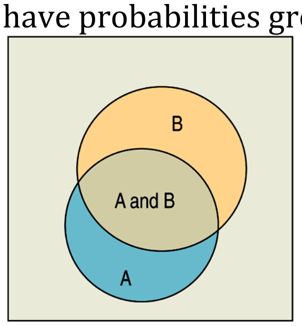
$$= \frac{3}{10} + \frac{5}{10} = \frac{8}{10} = \boxed{\frac{4}{5} \text{ or } .8 \text{ or } 80\%}$$

5. **Multiplication Rule**: For two independent events $A$ and $B$, the probability that both $A$ and $B$ occur is the product of the probabilities of the two events.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Generalize for more than 2 events.

Two independent events $A$ and $B$ are not disjoint, provided the two events have probabilities greater than 0.




Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

What Can Go Wrong?

- Beware of probabilities that do not add up to 1.
- Do not add probabilities of events that are not disjoint.
- Do not multiply probabilities of events if they are not independent.
- Do not confuse disjoint and independent.

p.335 (20)

| | 10% off | 20% off | 30% off | 50% off | |
|---|---|---|---|---|---|
| a) | .2 | .2 | .2 | .2 | Not legitimate since $.2+.2+.2+.2=.8 \neq 1$ |
| b) | .5 | .3 | .2 | .1 | Not legitimate since $.5+.3+.2+.1=1.1 \neq 1$ |
| * c) | .8 | .1 | .05 | .05 | $.8+.1+.05+.05=1$ Legitimate |
| d) | .75 | .25 | .25 | -.25 | $.75+.25+.25+-.25=1$ Not legitimate since probability can't be Negative. |
| * e) | 1 | 0 | 0 | 0 | $1+0+0+0=1$ Legitimate |

p.337 (40) $P(A)=.4$ $P(B)=.11$ $P(O)=.45$ $P(AB)=?$ $.04$

a) Individual Donor

1. $.4+.11+.45=.96$ $P(AB)=1-.96=.04$
2. $P(A \text{ or } B)=P(A)+P(B)=.4+.11=.51$
3. $P(\text{Not } O)=1-P(O)=1-.45=.55$ or 55%

b) Four Donors (Multiplication Rule)

1. $P(\text{All Type } O)=P(O)\times P(O)\times P(O)\times P(O)$
   $=.45\times.45\times.45\times.45=.45^4\approx.041$
2. $P(\text{None Type } AB)=P(AB^c)\times P(AB^c)\times P(AB^c)\times P(AB^c)$
   $=.96\times.96\times.96\times.96=.96^4\approx.849$
3. $P(\text{Not all Type } A)=1-P(\text{All Type } A)=1-.4^4=.9744$
   0, 1, 2, or 3 Type A
4. $P(\text{At least one Type } B)=1-P(\text{None } B)=1-.89^4\approx.373$
   1, 2, 3, or 4 Type B $P(B)=.11$ $P(\text{Not } B)=1-.11=.89$