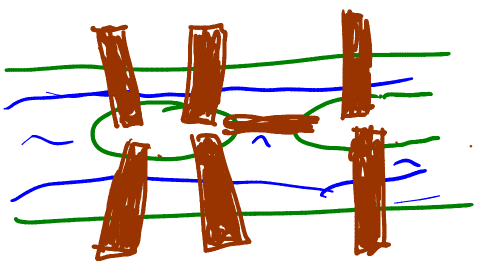
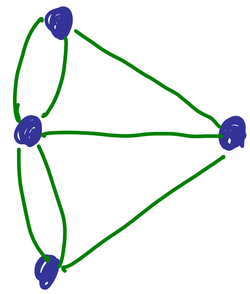
14.1 – Graphs, Paths, and Circuits

In the 18th century the townspeople of the Prussian Town of Königsberg wanted to see if they could find a path though town that would cross each of their bridges in town exactly once. A Swiss mathematician named Leonhard Euler published a paper about the problem. He replaced each piece of land with a point and the bridges with a curve.





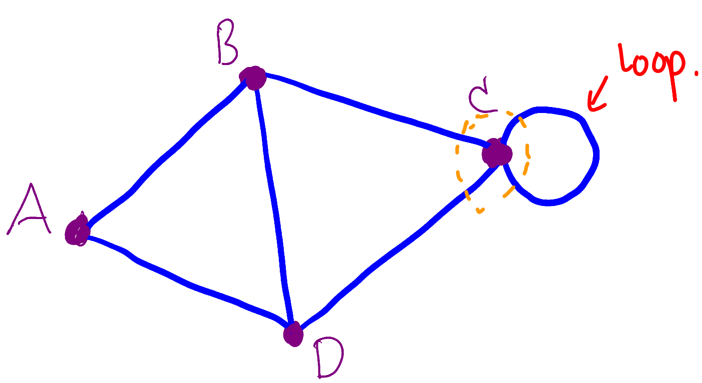
There are many situations where the study of graphs can be used in the Real World.

- Transportation like Airlines, Bus, or Train routes between cities.
- Delivery service like US Postal Service or UPS.
- Telecommunication or computer networks.
- Community planning snow removal or street cleaning.

Graph is a diagram that consists of a finite set of points called **vertices** that are connected by lines or curves called **edges**. (Single point is **vertex**)
Loop is an edge that starts and ends at the same vertex.
Adjacent Vertices are connected by at least one edge.

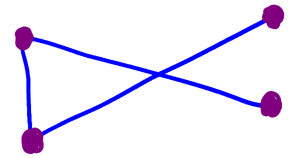
Graph Theory is the study of graphs and their applications.

The **Degree of a Vertex** is the number of edges at a vertex.
Odd Vertex – vertex with an odd number of edges attached to it.
Even Vertex – vertex with an even number of edges attached to it.



Vertex	# of Edges	
A	2	Even
B	3	odd
C	4	Even
D	3	odd

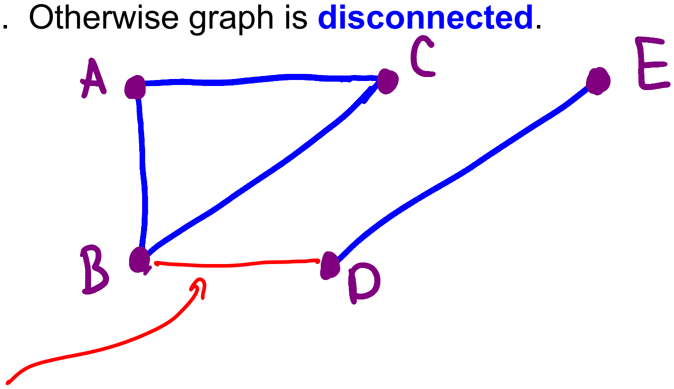
Vertices will always have dots.



Path is a route along edges that start at a vertex and end at a vertex.

Circuit is a path that begins and ends at the same vertex.

A graph is **connected** if for any two vertices there at least one path connecting them. Otherwise graph is **disconnected**.



Disconnected graph Since No Path From C to E or B to D, etc.

Bridge is an edge that if removed will result in a disconnected graph.

14.2 – Euler Paths and Euler Circuits

Euler Path is a path that includes every edge of a graph exactly once.
Euler Circuit is a circuit that includes each edge exactly once. Since a circuit it should begin and end at the same vertex.

Note: An Euler Circuit is always and Euler Path, but an Euler Path may not be an Euler Circuit.

Euler's Theorem

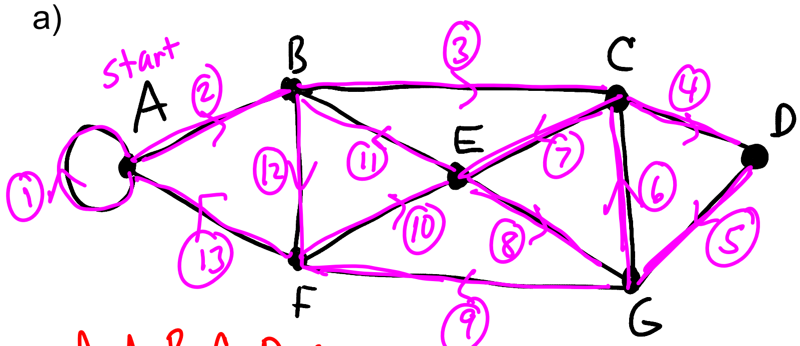
1. If a graph has exactly two odd vertices then it has at least one Euler Path but no Euler Circuit. Each Euler Path will begin at one of the odd vertex and end at the other one.

2. If a graph has all even vertices then it has at least one Euler Circuit (which is an Euler Path). The Euler Circuit can start and end at any vertex.

3. If a graph has more than two odd vertices, then it will have no Euler Circuits or Euler Paths.

Examples

a)

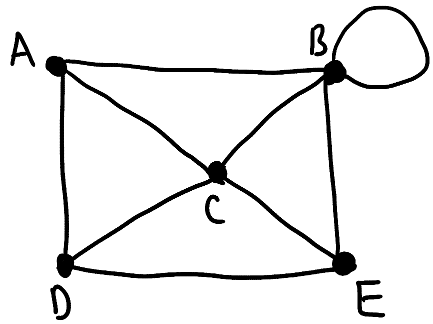


A, A, B, C, D, G, C, E, G, F, E, B, F, A

Vertex	# of Edges
A	4
B	4
C	4
D	2
E	4
F	4
G	4

All vertices are even so graph has an Euler Circuit (Euler Path).

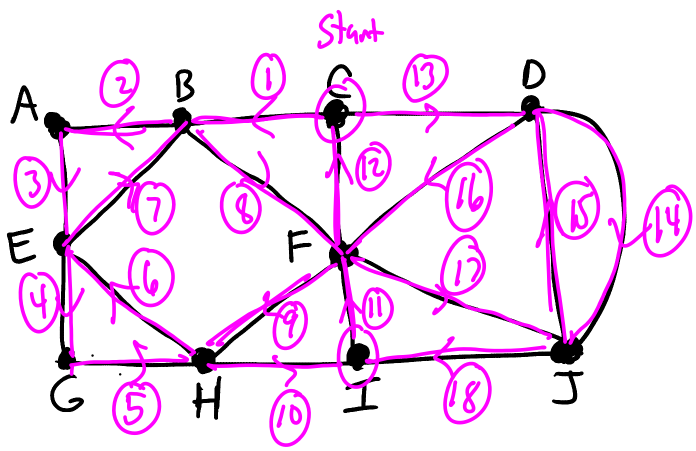
b)



vertex	# of Edges
A	3
B	5
C	4
D	3
E	3

Since more than 2 odd graph does not have Euler Path or Euler Circuit.

c)

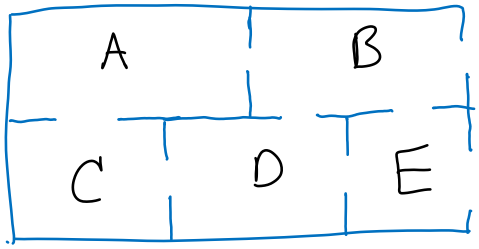


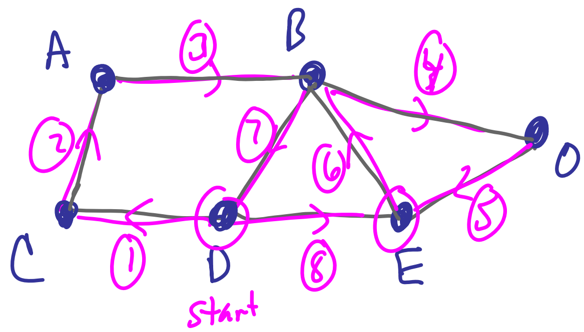
vertex	# of Edges
A	2
B	4
C	3
D	4
E	4
F	6
G	2
H	4
I	3
J	4

Graph has exactly 2 odd vertices so graph has Euler Path but no Euler circuit.

Floor plans

(See also p. 906, Example 3)

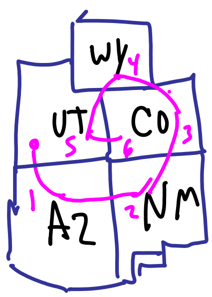




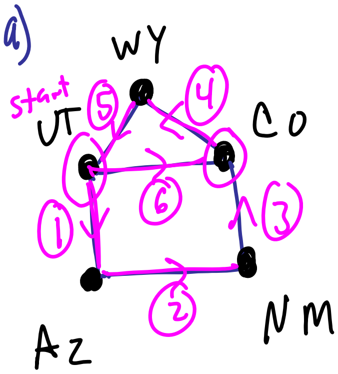
Vertex	Edges
A	2
B	4
C	2
D	3
E	3
O	2

Has Euler Path since Exactly 2 odds but No Euler Circuit.

p.913 #60



a)



b)

Vertex	Edges
WY	2
UT	3
CO	3
AZ	2
NM	2

Euler Path is Possible

d) Circuits Not Possible since there are exactly 2 odds

Neighborhoods