15.3 – Hamilton Paths and Circuits

In the last section we studied paths and circuits that covered every edge (Euler Circuits and Paths). In this section we will look at paths that visit each vertex exactly once without having to use each edge. An example would be a delivery person who must make deliveries to several locations. These types of paths were studied by the Irish mathematician, Sir William Rowan Hamilton. Therefore they are named after him.

Hamilton Path is a path that contains each vertex of a graph exactly once.

Hamilton Circuit is a circuit that begins at some vertex and goes through every vertex exactly once to return to the starting vertex.

Some books call these Hamiltonian Paths and Hamiltonian Circuits.

There is no easy theorem like Euler's Theorem to tell if a graph has Hamilton Circuit.

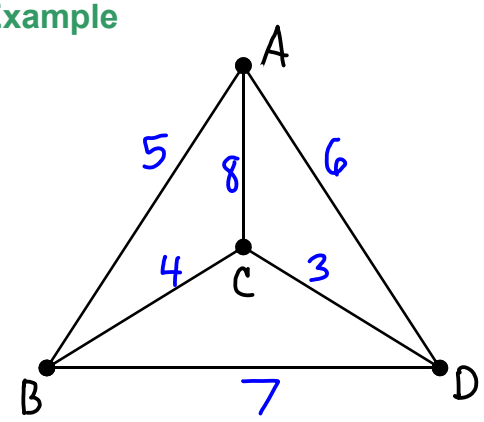
Examples
p. 849: #6 & #8

Weighted Graph is a graph with a "weight" assigned to each edge. The weight could represent distance, cost, etc.

Complete Graph is a graph with exactly one edge between each pair of vertices. Every Complete Graph of 3 or more vertices has a Hamilton Circuit.

The **Traveling Salesman Problem** is the problem of finding a Hamiltonian Circuit in a complete weighted graph for which the sum of the weights is a minimum.

Example

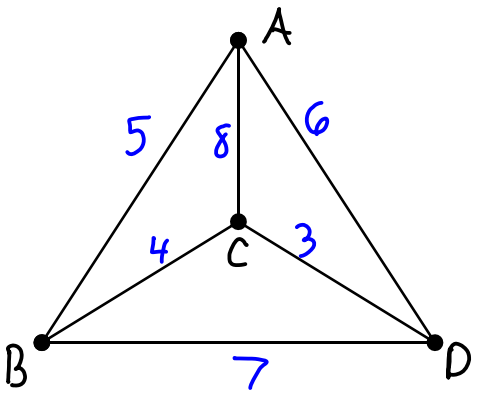


$k! = k \cdot (k-1) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$

If *n* = number of vertices then the total number of unique Hamiltonian Circuits for a complete graph is $\frac{(n-1)!}{2}$. The number of possible Hamiltonian Circuits increases greatly as the number of vertices increases. Even with computers it would be impossible to solve if *n* is too large. (p. 847)

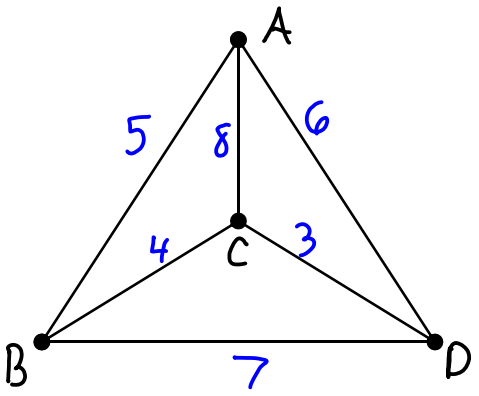
Nearest Neighbor Algorithm

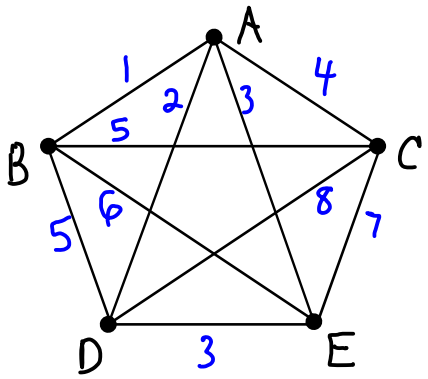
1. Choose a starting vertex.
2. The next vertex chosen is the one "nearest" (smallest weight) that has not been visited. If tie choose one. Mark the edges covered as they are added to the path.
3. If all vertices have not been visited then repeat step 2 otherwise return to the start.

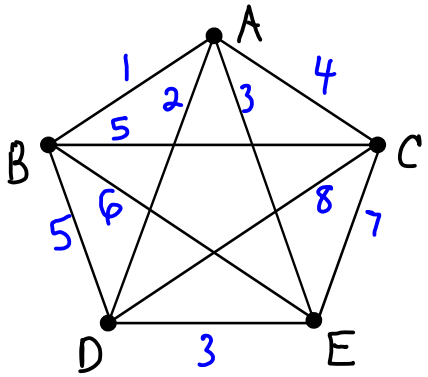


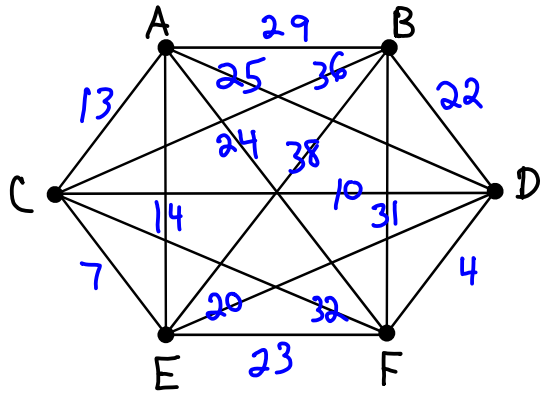
Greedy Algorithm (Sorted Edges)

1. List edges in increasing order
2. Choose edge of least weight and mark edge.
3. Choose next edge of least weight if the edge does not complete a circuit before all vertices have been visited OR does not result in 3 marked edges at a vertex.
4. Continue step 3 until Hamiltonian Circuit is formed.





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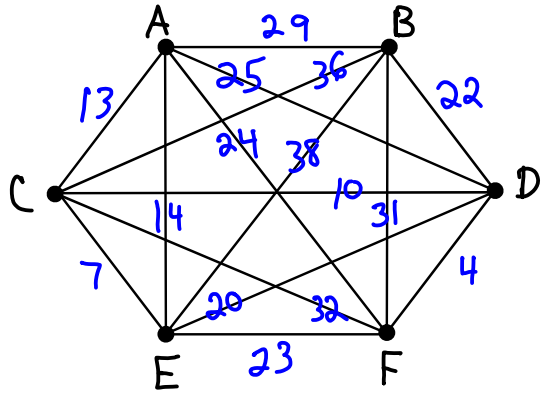
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