15.2 – Euler Paths and Euler Circuits

Euler Path is a path that includes every edge of a graph exactly once.
Euler Circuit is a circuit that includes each edge exactly once. Since a circuit it should begin and end at the same vertex.

Note: An Euler Circuit is always and Euler Path, but an Euler Path may not be an Euler Circuit.

Euler's Theorem

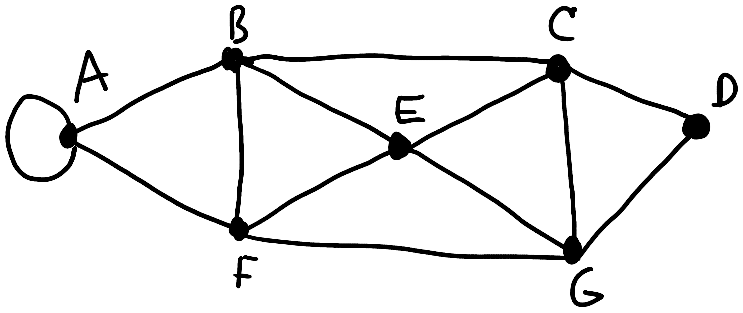
1. If a graph has exactly two odd vertices then it has at least one Euler Path but no Euler Circuit. Each Euler Path will begin at one of the odd vertex and end at the other one.

2. If a graph has all even vertices then it has at least one Euler Circuit (which is an Euler Path). The Euler Circuit can start and end at any vertex.

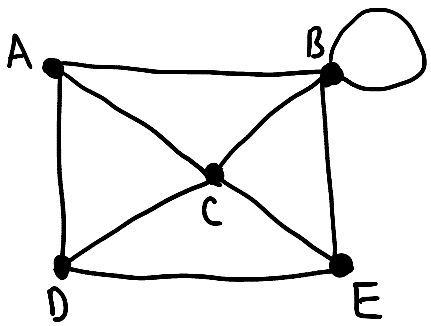
3. If a graph has more than two odd vertices, then it will have no Euler Circuits or Euler Paths.

Examples

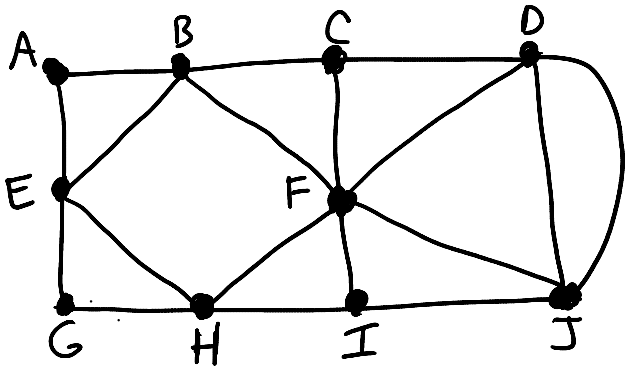
a)



b)

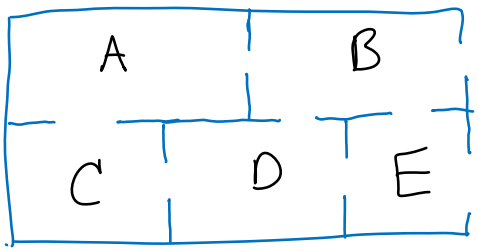


c)



Floor plans

(See also p. 834, Example 3)



Neighborhoods