Calculus I

Review Exam 5

Sections 4.5, 4.7, 5.1, 5.2, 5.3, 5.4

§4.5. Optimization. These sections are applications of the derivative to optimization. The general problem will ask us to optimize (minimize or maximize) some quantity subject to a restriction of some kind. (1) We first locate in the problem statement the quantity to be optimized; (2) we then draw any relevant pictures, assign any necessary variables and develop a rough equation for the quantity to be optimized; (3) If there is more than one independent variable, we look in the problem statement for the restriction and find in this restriction an equation relating the independent variables. We use this equation to substitute until we have the quantity to be optimized expressed in terms of a single independent variable; (4) we look for an interval on which this equation is naturally defined; (5) We find the absolute extrema of this function on the given interval.

1. Robin has 1200 yards of fencing. With it, she intends to make two pens by making a large rectangular enclosure with a fence down the middle parallel to one pair of sides. What are the dimensions of the pen with the maximum area?

2. Now suppose Robin has 1600 yards of fencing. With it, she intends to make three pens by making a large rectangular enclosure with two fences down the middle, parallel to one pair of sides. What are the dimensions of the pen with the maximum area?

3. A sheet of paper is to contain 18 square inches of print. The margins at the top and bottom are each 2 inches. The right and left margin are to be 1 inch each. What are the dimensions of the sheet of paper if the total area of the paper is to be a minimum?

4. Suppose that the daily cost function for a firm is given by C(x) = 1250 + 50x + .5x 2 . Find the production level which will minimize the average cost.

5. A firm which produces mini microwaves has determined that the daily demand function for its microwaves is given by 150 ? 0.2x for 0 ? x ? 750.

(a) Find the daily revenue function.

(b) Suppose that the daily cost function is given by: C(x) = 1000 + 50x + 0.05x 2 . Find the daily profit function.

(c) Find the production level which will maximize the daily profit.

§4.7. Antiderivatives. This section holds the concept of reversing the derivative. With the exception of the power rule, each derivative formula is interpreted in reverse to find an antiderivative. The antidifferentiation formula for power functions looks somewhat different: $\frac{d}{dx}[x^n] = \frac{x^{n+1}}{n+1}$, but still reverses the derivative of a power function.

6. (a) Find the most general antiderivative of $f'(x) = 3\sqrt{x} + \dfrac{1}{x} - 3\sec^2(x) + e^x$

(b). Find the particular $f(x)$ which satisfies (a), if $f(1) = e$.

7. Find the most general antiderivative of the function: $f(x) = x \cdot \sqrt{x} - \cos(x) + \dfrac{1}{x}$.

8. Find f if $f''(x) = e^x - 3$, $f'(0) = 1$, $f(0) = -1$.

9. Find the most general antiderivative of $f(x) = \dfrac{x^2 + 3xe^x - 4}{x}$.

10. Find the most general antiderivative of $f'(x) = \dfrac{x^3 + x + 2}{x^2} - 3\sec(x)\tan(x)$.

11. Find f if $f''(x) = 2e^x + 2x$, $f'(0) = 1$, $f(0) = -1$.

12. Find the most general antiderivative of $f(x) = x(x^3 - 2\dfrac{e^x}{x} - 2)$.

§**5.1. Areas and Distances:** We begin estimating the area between a curve and the x-axis on an interval, and also to assign an interpretation to that number

13. Approximate the area between the graph of $y = f(x) = 5x - x^2$ and the x–axis on the interval $[0, 4]$ using R_4. Using L_4. Using M_4. Compute R_n. Compute $\lim_{n\to\infty} R_n$.

14. Find R_N, for the function $f(x) = 4x - x^2$ on the interval $[1, 3]$. Compute $\lim_{n\to\infty} R_n$.

15. The velocity of a particle at time t is given by $v(t) = 3t + 2$. Find the area between the graph of this function and the t axis from $t = 0$ and $t = 3$. [Just sketch the picture and find the exact area (since it is a trapezoid); don't bother with an approximation.]

§**5.2. The Definite Integral:** In this section we define the Definite Integral as the limit of Riemann Sums, and investigate its properties.

16. Write the definite integral $\displaystyle\int_{-1}^{3} \sin(x^2)\, dx$ as a limit of Riemann sums.

17. Write the limit of Riemann sums: $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} x_k \cos(x_k)\Delta x$ on $[0, \pi]$ as a definite integral.

18. Compute the definite integral $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\, dx$ by considering the definition of the symbol itself and interpreting it in terms of areas. Do not compute the limit of any Riemann sums or the FTOC.

19. Compute the definite integral $\displaystyle\int_{0}^{5} 7 - 2x\, dx$ by considering the definition of the symbol itself and interpreting it in terms of areas. Do not compute the limit of any Riemann sums or the FTOC.

20. Express the area between the graph of $f(x) = 9 - x^2$ and the x–axis on the interval $[-3, 3]$ as both a definite integral, and a limit of Riemann sums. Make no attempt to compute either.

21. Suppose that $\displaystyle\int_{-2}^{3} 3x^2 = 35$ and $\displaystyle\int_{-2}^{1} 3x^2\, dx = 26$. Determine $\displaystyle\int_{1}^{3} 3x^2\, dx$.

22. Suppose that $\displaystyle\int_{1}^{5} f(x)\, dx = 7$ and $\displaystyle\int_{-2}^{1} f(x)\, dx = 11$. Determine $\displaystyle\int_{-2}^{5} f(x)\, dx$.

23. Suppose that $\displaystyle\int_{0}^{3} x^2\, dx = 9$, $\displaystyle\int_{0}^{3} x\, dx = 9/2$, and $\displaystyle\int_{0}^{3} 1\, dx = 3$.
(a) Find $\int_0^3 (2x^2 - 3x + 7)\, dx$. (b) Find $\int_3^0 4x^2\, dx$. (c) Find $\int_0^3 (4 - 5x + 3x^3)\, dx$.

§**5.3. Evaluating Definite Integrals:**

24. Compute the following definite integrals [using the Fundamental Theorem of Calculus, Part 1].

(a) $\displaystyle\int_{-2}^{1} 2x - \cos(x)\, dx$ (b) $\displaystyle\int_{1}^{3} 2x(x^2 + 1)dx$

(c) $\displaystyle\int_{1}^{3} \left(\frac{x^3 + 1}{x}\right)\, dx$ (d) $\displaystyle\int_{0}^{1} (\sin(x) + 2\sec^2(x))\, dx$

(e) $\int_0^4 x\sqrt{x}\, dx$ (f) $\displaystyle\int_{0}^{\pi/4} (\sec(x)\tan(x) + 2\sin(x))\, dx$

25. Find the general indefinite integral: (a) $\int x(2x + 3)^2\, dx$ (b) $\displaystyle\int \frac{x^2 + \sqrt{x}}{\sqrt{x}}\, dx$ (c) $\displaystyle\int \frac{\sec(x)}{1 + \tan^2(x)}\, dx$

26. A honeybee population starts with 500 bees and increases at the rate of $p'(t)$ bees per week. What does $500 + \int_0^{15} p(t)\, dt$ represent?

§**5.4. The Fundamental Theorem of Calculus:**

27. Use the Fundamental Theorem of Calculus to compute:

(a) $\displaystyle\frac{d}{dx}\left[\int_{1}^{x^2} \tan(t^2)\, dt\right]$ (b) $\displaystyle\frac{d}{dx}\left[\int_{1}^{x} \sin(t^2)\, dt\right]$ (c) $\displaystyle\frac{d}{dx}\left[\int_{1}^{x^3} t\ln(t)\, dt\right]$

28. Find the average value of the function $f(x) = 2x^3 + 2x$ on the interval $[1, 3]$.