Calculus and Analytic Geometry I
Exam 4–Review
Sections 3.6, 3.7, 4.1, 4.2, 4.3, 4.4, 4.5

§**3.6. Hyperbolic Functions** We received 6 new functions with their definitions (the hyperbolic trigonometric function) and their 6 respective inverse functions, along with the differentiation formulas for each. You should be able to make basic computations with them and limits of functions containing them, as well as compute their derivatives.

For Review, look at §3.6 on pages 189-190: 3, 5, 11,14, 17, 19bdef, 27, 29, 32,35, 36, 37, 38, 39.

§**3.7. L'Hôpital's Rule** We looked at limits early in the course and learned how to interpret some forms (for instance, $\frac{c}{0}$, and some $\frac{0}{0}$ forms), but there are 7 forms which we need more machinery to interpret. These are the $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^∞, ∞^0, and 0^0 forms. L'Hôpital's Rule helps us deal with the first two. For the third form, we physically subtract the two to get a fractional form (likely one of the first two) which we can interpret in some way. The fourth form we turn into a fractional form by noting that $a \cdot b = \frac{b}{1/a}$; this gives us a fractional form which we can likely apply L'Hôpital's Rule to. The last three forms we can deal with by noting that $a^r = e^{r\ln(a)}$. The limit of $r\ln(a)$ gives us a form of type 4, which we can then deal with in the standard way. Below are 10 examples.

Compute the following limits. Be careful: some of the them are not indeterminate (that is, they can be interpreted without applying any special techniques).

1. $\displaystyle\lim_{x\to\infty} \frac{x}{e^{-x}}$
2. $\displaystyle\lim_{x\to\infty} x\ln(x)$
3. $\displaystyle\lim_{x\to 0^+} (\sin(x))^{1/x}$
4. $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{e^x}$
5. $\displaystyle\lim_{x\to\pi/2^-} \frac{\tan(x)}{x-\pi/2}$
6. $\displaystyle\lim_{x\to\infty} \sqrt{x}-\sqrt{x-1}$
7. $\displaystyle\lim_{x\to-\infty} xe^{-x}$
8. $\displaystyle\lim_{x\to 0} \frac{1-e^{-x}}{\ln(x+1)}$
9. $\displaystyle\lim_{x\to 0} x^{\sin(x)}$
10. $\displaystyle\lim_{x\to\infty} (\ln(x))^{1/x}$

§**4.1. Maximum and Minimum Values:** This section introduced us to a lot of terminology: Critical Numbers, Absolute Maximum & Local Maximum, Absolute Minimum & Local Minimum, and several important theorems: The Extreme Value Theorem and Fermat's Theorem.

1. Find the critical numbers of the function: $f(x) = \dfrac{x}{4} + \dfrac{1}{x}$.

2. Find the absolute maximum and the absolute minimum of the function $f(x) = \dfrac{x}{4} + \dfrac{1}{x}$ on the interval $[1/2, 4]$.

3. Work other examples of this type. They are all basically the same, just with different functions and derivatives.

4. Also be able to distinguish in a graph the difference between a local extrema and an absolute extrema.

§**4.2: The Mean Value Theorem**

1. Verify that the hypotheses of the Mean Value Theorem are satisfied for $f(x) = x^3 - 4x + 2$ on the interval $[-1, 2]$ and find the value(s) of c provided by its conclusion.

2. Verify that the hypotheses of Rolle's Theorem are satisfied for $f(x) = \sqrt{x} - \frac{x}{2}$ on $[0, 4]$ are satisfied and find the value(s) of c provided by its conclusion.

3. Be able to graphically interpret the MVT as in exercise 7 on page 215.

§**4.3 and 4.4: Derivatives, shapes of graphs, and curve sketching.**

1. Let $f(x) = \dfrac{3}{5}x^5 - 4x^3 + 8$.
   (a) Find the intervals on which the function f is increasing and the intervals on which the function f is decreasing.
   (b) Find any local maxima and local minima of the function f.
   (c) Find the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.
   (d) Find any inflection points of the graph of f.
   (e) Use the data you found in parts (a)–(d) to sketch the graph of the function f.

2. Let $f(x) = \left(\dfrac{x-1}{x+1}\right)^2$; note that $f'(x) = \dfrac{4(x-1)}{(x+1)^3}$ and $f''(x) = \dfrac{-8(x-2)}{(x+1)^4}$. Keep me honest by double-checking my derivatives. (a) Find the domain of f; (b) find any vertical asymptotes for f; (c) find any horizontal asymptotes for the function f; (d) find the intervals on which f is increasing, and the intervals on which f is decreasing. (e) find any local extrema; (f) find the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down; (g) find any inflection points; (h) find the x–intercepts of the function f; (i) find the y–intercept of the function f; (j) Use this information to sketch a graph of this function.

3. Let $f(x) = \left(\dfrac{x-1}{x+3}\right)^2$; note that $f'(x) = \dfrac{8(x-1)}{(x+3)^3}$ and $f''(x) = \dfrac{-16(x-3)}{(x+3)^4}$. Keep me honest by double-checking my derivatives. (a) Find the domain of f; (b) find any vertical asymptotes for f; (c) find any horizontal asymptotes for the function f; (d) find the intervals on which f is increasing, and the intervals on which f is decreasing; (e) find the x–intercepts of the function f; (f) find the y–intercept of the function f; (g) Use this information to sketch a graph of this function.

4. Suppose that $f(x) = \dfrac{2x-3}{(x-2)}$. (a) Find the domain of f; (b) find any vertical asymptotes for f; (c) find any horizontal asymptotes for the function f; (d) find the intervals on which f is increasing, and the intervals on which f is decreasing; (e) find any local extrema; (f) find the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down; (g) find any inflection points; (h) find the x–intercepts of the function f; (i) find the y–intercept of the function f; (j) Use this information to sketch a graph of this function.

§**4.5. Optimization.** These sections are applications of the derivative to optimization. The general problem will ask us to optimize (minimize or maximize) some quantity subject to a restriction of some kind. (1) We first locate in the problem statement the quantity to be optimized; (2) we then draw any relevant pictures, assign any necessary variables and develop a rough equation for the quantity to be optimized; (3) If there is more than one independent variable, we look in the problem statement for the restriction and find in this restriction an equation relating the independent variables. We use this equation to substitute until we have the quantity to be optimized expressed in terms of a single independent variable; (4) we look for an interval on which this equation is naturally defined; (5) We find the absolute extrema of this function on the given interval.

3. Robin has 1200 yards of fencing. With it, she intends to make two pens by making a large rectangular enclosure with a fence down the middle parallel to one pair of sides. What are the dimensions of the pen with the maximum area?

4. Now suppose Robin has 1600 yards of fencing. With it, she intends to make three pens by making a large rectangular enclosure with two fences down the middle, parallel to one pair of sides. What are the dimensions of the pen with the maximum area?

5. A sheet of paper is to contain 18 square inches of print. The margins at the top and bottom are each 2 inches. The right and left margin are to be 1 inch each. What are the dimensions of the sheet of paper if the total area of the paper is to be a minimum?

6. Suppose that the daily cost function for a firm is given by $C(x) = 1250 + 50x + .5x^2$. Find the production level which will minimize the average cost.

7. A firm which produces mini microwaves has determined that the daily demand function for its microwaves is given by $p = 150 - 0.2x$ for $0 \leq x \leq 750$.

(a) Find the daily revenue function.

(b) Suppose that the daily cost function is given by: $C(x) = 1000 + 50x + 0.05x^2$. Find the daily profit function.

(c) Find the production level which will maximize the daily profit.