Calculus and Analytic Geometry I
Exam 2—Sections 1.6—2.6, inclusive

Limits as $x \to \infty$

Section 1.6: Be able to compute limits as x goes to ∞. Examples:

(a) $\displaystyle\lim_{x\to\infty} \frac{2x-4x^2}{2x^2+3}$ (b) $\displaystyle\lim_{x\to\infty} \frac{2x-4x^3}{2x^2+3}$ (c) $\displaystyle\lim_{x\to\infty} \frac{2x-4}{2x^2+3}$

(d) $\displaystyle\lim_{x\to\infty} \sqrt{x+1}-\sqrt{x}$ (e) $\displaystyle\lim_{x\to-\infty} \frac{2x-3}{3x+\sqrt{4x^2+1}}$ (f) $\displaystyle\lim_{x\to\infty} \frac{2x-3}{3x+\sqrt{4x^2+1}}$

Origins of the Derivative

Section 2.1, 2.2: Be able to answer questions regarding the origins of the derivative. Examples

(a) $\displaystyle f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ (b) Compute: $\displaystyle f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(c) f differentiable $\Rightarrow$ f continuous, smooth (d) graphical analysis of differentiability as in homework (below).

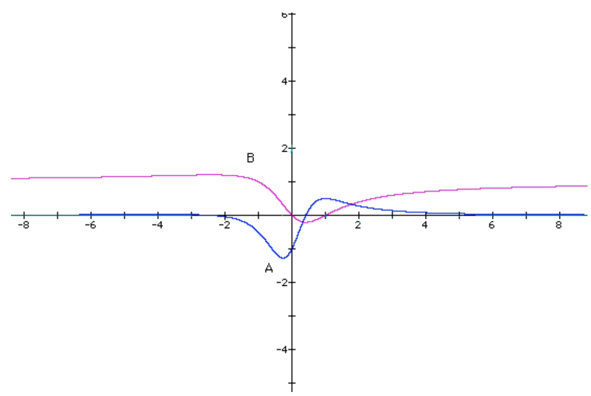
(e) $f(x) = x^4 - 2x^2 + 1$. Find all points on the graph where the tangent line is horizontal.

(f) $f(x) = \dfrac{1}{x-1}$. Find an equation for the line tangent to the graph of f at the point $(2, 1)$.

(g) $f(x) = x^3 - 3x^2 + 1$. Find all points on the graph where the tangent line has slope 9. Find an equation for each.

(from b) For $f(x) = \sqrt{x}$, $f(x) = 3x - 2x^2 - 1$, $f(x) = \frac{x}{x-1}$
Compute $f'(x)$ using the definition as in (b).

(from d) Determine which is the graph of f and which is the graph of f'. Sketch the graph of f''.



Applications of the Derivative

Section 2.2: Be able to answer questions regarding applications of the derivative. Examples:

(a) A rock is thrown vertically upward from ground level with an initial velocity of 96 ft/sec. The position of the rock t seconds after it is released is given by: $s(t) = -16t^2 + 96t$. What is the velocity of the rock after 3 seconds? When is the rock at its maximum height? What is the maximum height of the rock? When will it hit the ground? With what velocity will it hit the ground?

(b) Air is blown into a spherical balloon. What is the rate of change of the volume of air in the balloon with respect to its radius when $r = 4$ inches? Interpret this.

(c) The daily cost of producing x bicycles is given by: $C(x) = 9000 + 250x + 0.005x^2$. What is the rate of change of cost with respect to daily production level when $x = 20$? Interpret this.

Computation of the Derivative.

Sections 2.3 (atoms, +, -), 2.4 ($\cdot$, $\div$), 2.5 (chain rule), 2.6 (implicit differentiation). Be able to compute any derivative thrown at you. Examples: differentiate the following functions, or answer the posed question.

(a) $f(x) = 5x^3 - 2x\sqrt[3]{x^2} - \frac{7}{x^3}$ (b) $y = \sin(x) - 4\tan(x) + 2\csc(x)$

(c) Find an equation of the tangent line to the curve $y = x^2 - 4x$ which is perpendicular to the line $x + 2y = 7$.

(d) Find the second derivative of the function $y = \sin(x) - 4\tan(x) + 2\csc(x)$.

(e) $f(x) = x^2\sin(x) + \dfrac{\cos(x)}{x^2+1}$ (f) $s = \dfrac{t\sec(t)}{1+\sin(t)}$

(g) Find an equation of the line tangent to the curve $y = \dfrac{\sqrt{x}}{x+1}$ at the point $(4, 2/5)$ on the curve.

(h) $f(x) = \sin(x^2+1)$ (i) $g(x) = \sin^{10}(x^2+1)$

(j) $y = \sin^{10}(x^2\tan(x))$ (k) $y = \sin^{10}\left(\dfrac{\cos(x)}{x^2+1}\right)$

(l) $g(x) = \sec(\sqrt{x}\tan(x))$

(m) $y = \sec(\sqrt{x^2+1})$

(n) $f(x) = \cos(x^2\sqrt{x^2+1})$

(o) $y = \cos^3(x^2\sqrt{x^2+1})$

(p) Compute: $s = \dfrac{5t}{\sqrt[3]{3t+1}}$ and simplify the derivative to get $\frac{ds}{dt} = \frac{5(2t+1)}{(3t+1)^{4/3}}$.

(q) Find $\frac{dy}{dx}$ by implicit differentiation: $y^4 - 3x^2y^3 = 2 - 5x^3y$

(r) Find $\frac{dy}{dx}$ by implicit differentiation: $\sin(xy) = 1 + x\cos(y)$

(s) Find $\frac{dy}{dx}$ by implicit differentiation: $2x^2y - \sqrt{xy} = 6$.

(t) Verify that the point $(1, 2)$ is on the curve: $y^4 - 3x^2y^3 = 2 - 5x^3y$. Find an equation for the line tangent to the curve $y^4 - 3x^2y^3 = 2 - 5x^3y$ at the point $(1, 2)$. [Note that you have already computed its implicit derivative in (q).]

Below is a graph of the implicit function in (q) and (t), together with the tangent line.