Calculus I—Review Exam 1

This exam will discuss some algebra review and all of Chapter 1.

Review material:

1. Expand: $(2x-3y)^2$

2. Factor: $4x^{4/3} - 20x^{1/3} + 16x^{-2/3}$

3. Factor: $3x^3(x^2+2)^{-3/2} + 2x(x^2+2)^{-1/2}$

4. Rationalize: $\dfrac{h}{\sqrt{h+9}-3}$

5. Solve: $x^2 = 4x$

6. Solve: $(x-2)^{3/2} = 8$

7. Find the equation of the line through $(1,-2)$ and perpendicular to $2x+4y=7$

8. Simplify: $\dfrac{\frac{1}{x}-\frac{1}{y}}{x+y}$

9. For $f(x) = 3x-5-2x^2$, compute $\dfrac{f(a+h)-f(a)}{h}$

10. For $f(x) = \dfrac{x}{x-1}$ and $g(x) = 2x+5$, compute $(f\circ g)(x)$ and $(g\circ f)(x)$.

11. For $h(x) = (\sin(x)+\ln(x))^3$, find functions f and g so that $h(x) = (f\circ g)(x)$

12. Without a calculator, compute $\tan(\pi/6)$, $\tan(5\pi/6)$, $\tan(7\pi/6)$, and $\tan(11\pi/6)$.

13. What do all those angles have in common? What do all their tangents have in common? Be able to convert between degrees and radians.

14. Find all solutions to the equation: $2\sin^2(x) - \sin(x) = 1$. without using a calculator.

15. Find the domain of the functions: $f(x) = \dfrac{x-1}{\sqrt{2x+5}}$.

Calculus:

1. Consider the graph at right of the function $f(x)$.
Find the following:

a) $\lim\limits_{x\to 1^-} f(x) =$

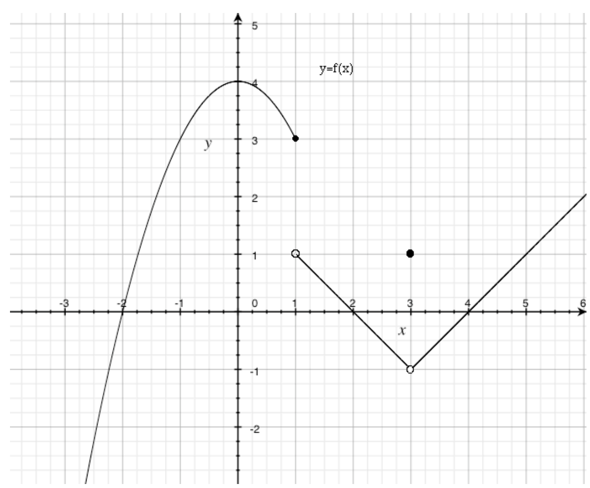
b) $\lim\limits_{x\to 1^+} f(x) =$

c) $\lim\limits_{x\to 1} f(x) =$

d) $f(3) =$

e) $\lim\limits_{x\to 3^-} f(x) =$

(f) $\lim\limits_{x\to 3} f(x) =$



3. For (a) and (g), first estimate the limit (if you think the limit exists) by substituting appropriate numerical values. Compute the limits when they exist. Whenever one of the limits below does not exist, state why.

(a) $\lim\limits_{x\to 2^-} \dfrac{x-6}{x-2} =$

(b) $\lim\limits_{x\to 2^+} \dfrac{x-6}{x-2} =$

(c) $\lim\limits_{x\to 2} \dfrac{x-6}{x-2} =$

(d) $\lim\limits_{x\to 2^-} \dfrac{x+2}{(x-2)^2} =$

(e) $\lim\limits_{x\to 2^+} \dfrac{x+2}{(x-2)^2} =$

(f) $\lim\limits_{x\to 2} \dfrac{x+2}{(x-2)^2} =$

(g) $\lim\limits_{x\to 1} \dfrac{x^2-x-6}{x-3} =$

(h) $\lim\limits_{x\to 3^+} \dfrac{x^2-x-6}{x-3} =$

(i) $\lim\limits_{x\to 16} \dfrac{\sqrt{x}-4}{x-16} =$

4. Compute the following limits.

(a) $\lim\limits_{x\to 0} \dfrac{\sin(x)\tan(3x)}{x^2}$

(b) $\lim\limits_{x\to 0} \dfrac{2x}{\sin(4x)}$

(c) $\lim\limits_{x\to 1} \dfrac{x-1}{\sin(x-1)}$

(d) $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{x}$

(e) $\lim\limits_{x\to 0} \dfrac{\cot(4x)}{x}$

(f) $\lim\limits_{x\to 0} \dfrac{\sin(x)-\sin(x)\cos(x)}{3x}$

5. Suppose that the function $f(x)$ is given by: $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2; \\ 0 & \text{if } x = 2; \\ x - 1 & \text{if } x > 2. \end{cases}$

Compute the following limit, if it exists. If it does not exist, state why. $\lim_{x \to 2} f(x) =$

6. Let $f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & \text{if } x \neq 4; \\ 8 & \text{if } x = 4. \end{cases}$

(a) Is f continuous at $x = 4$? Why or why not?

(b) Is f continuous on $(-\infty, 4) \cup (4, \infty)$? Why or why not?

7. (a) Where is the function $f(x) = \sqrt{25 - x^2}$ continuous?

(b) Where is the function $g(x) = \dfrac{\sin(x)}{x^2 - x - 2}$ continuous?

8. Explain why $f(x) = \sin((x^2 - x))$ is continuous on $(-\infty, \infty)$ and then use continuity to evaluate the following limit: $\lim_{x \to 1} \sin((x^2 - x))$. [i.e. If a function is continuous at $x = a$, what is $\lim_{x \to a} f(x)$. Explain.

9. Use the intermediate value theorem to show that there is a solution to the equation $f(x) = x^5 - x - 2 = 0$ somewhere in the interval $[1, 2]$.

10. Use the intermediate value theorem to solve the inequality: $9 - x^2 > 0$.

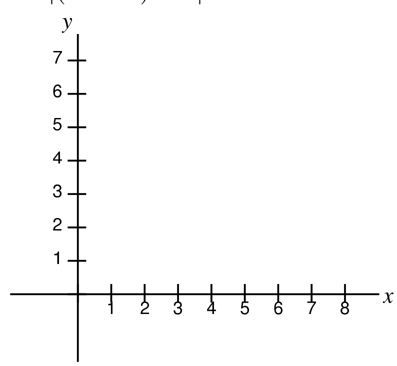
11. Write the $\delta - \epsilon$ definition of the following statement: $\lim_{x \to 2} 5x - 7 = 3$.

12. The $\delta - \epsilon$ definition of the following statement: $\lim_{x \to 3} 4x - 7 = 5$ **is**

For each $\epsilon > 0$, *there is a* $\delta > 0$ *such that if* $0 < |x - 3| < \delta$, *then* $|(4x - 7) - 5| < \epsilon$.

Prove this statement.

13. Graph the function $f(x) = 4x - 7$ on the graph at right, and then interpret the $\delta-\epsilon$ definition of the limit from question 12.



14. State in your own words what it means to say that $\lim_{x \to 3} 4x - 7 = 5$.

15. Compute the following limits involving infinity.

(a) $\lim_{x \to \infty} \dfrac{5 - 2x^2}{x^2 + 1} =$ (b) $\lim_{x \to \infty} \dfrac{x - 6}{x^2 + 1} =$ (c) $\lim_{x \to \infty} \dfrac{x^2 + 1}{3 - 2x} =$

(d) $\lim_{x \to \infty} \dfrac{x + 2}{\sqrt{x^2 + 1}} =$ (e) $\lim_{x \to \pi^-} x \cdot \csc(x) =$ (f) $\lim_{x \to \infty} \cos(x) =$

(g) $\lim_{x \to \infty} (\sqrt{x + 1} - \sqrt{x}) =$ (h) $\lim_{x \to -\infty} \dfrac{2x + 3}{5 - x} =$ (i) $\lim_{x \to \infty} \dfrac{\cos(x)}{x} =$