Calculus I—Review Final, Part 1

With respect to the review material on algebra and functions:

(1) Be able to solve algebraic, trigonometric, exponential, and logarithmic functions.

(2) Be able to compute the values of functions and inverse functions from a given graph of a function. So, given a graph of a function, be able to answer questions like: What is $f(2)$? and For which x is $f(x) = 3$.

(3) Be able to determine the domain and range of any of our basic functions (polynomials, rational functions, root functions, trigonometric functions, logarithmic and exponential functions) and compositions thereof. What is the domain of $f(x) = x^2 - x^3 + 3$, $f(x) = \dfrac{x^2-1}{x^2-3x+2}$, $f(x) = \sqrt{3-2x}$, $f(x) = \tan(2x)$, $f(x) = \ln(x+1)$, $f(x) = e^{x-1}$, $f(x) = e^{\tan(x)}$, $f(x) = \tan(e^x)$.

(4) Be able to compose functions and decompose functions. For $f(x) = x^2+1$, and $g(x) = \frac{1}{x-1}$, find $(f\circ g)(x)$, and $(g \circ f)(x)$. For $h(x) = \sqrt{x^2+1}$, find functions f and g such that $h(x) = (f \circ g)(x)$.

(5) Be able to use e^x and $\ln(x)$ as inverse functions of one another. Know the relationship between a function and its inverse.

Calculus Material

1. Suppose that a rock thrown vertically upward into the air with an initial velocity of 96 feet per second from the top of a building 112 feet high. Assuming no air resistance and that the acceleration due to gravity is $-32\text{–}f/s^2$, show that the position of the rock t seconds after the rock is released is given by $s(t) = 96t+112-16t^2$. (a) Find the instantaneous velocity of the rock at the time $t = 2$ seconds. (b) When will the rock reach its maximum height? (c) What is the maximum height of the rock? (d) When will the rock strike the ground at the base of the building? (e) What is the velocity of the rock as it strikes the ground?

2. Compute the limits when they exist. Whenever one of the limits below does not exist, state why. If the limit is an indeterminate form, use the technique from early in the course, as well as L'Hôpital's Rule when it can be effective.s

(a) $\displaystyle\lim_{x\to 2^-} \frac{x-6}{x-2} =$ (b) $\displaystyle\lim_{x\to 2^+} \frac{x-6}{x-2} =$ (c) $\displaystyle\lim_{x\to 2} \frac{x-6}{x-2} =$

(d) $\displaystyle\lim_{x\to 2^-} \frac{x+2}{(x-2)^2} =$ (e) $\displaystyle\lim_{x\to 2^+} \frac{x+2}{(x-2)^2} =$ (f) $\displaystyle\lim_{x\to 2} \frac{x+2}{(x-2)^2} =$

(g) $\displaystyle\lim_{x\to 1} \frac{x^2-x-6}{x-3} =$ (h) $\displaystyle\lim_{x\to 3^+} \frac{x^2-x-6}{x-3} =$ (i) $\displaystyle\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$

(j) $\displaystyle\lim_{x\to 0} \frac{\sin(x)\tan(3x)}{x^2}$ (k) $\displaystyle\lim_{x\to 0} \frac{2x}{\sin(4x)}$ (l) $\displaystyle\lim_{x\to 1} \frac{x-1}{\sin(x-1)}$

(m) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{x}$ (n) $\displaystyle\lim_{x\to 0} \frac{\cot(4x)}{x}$ (o) $\displaystyle\lim_{x\to 0} \frac{\sin(x)-\sin(x)\cos(x)}{3x}$

(p) $\displaystyle\lim_{x\to\infty} \frac{2x-4x^2}{2x^2+3}$ (q) $\displaystyle\lim_{x\to\infty} \frac{2x-4x^3}{2x^2+3}$ (r) $\displaystyle\lim_{x\to\infty} \frac{2x-4}{2x^2+3}$

(s) $\displaystyle\lim_{x\to\infty} \sqrt{x+1}-\sqrt{x}$ (t) $\displaystyle\lim_{x\to-\infty} \frac{2x-3}{3x+\sqrt{4x^2+1}}$ (u) $\displaystyle\lim_{x\to\infty} \frac{2x-3}{3x+\sqrt{4x^2+1}}$

(v) $\displaystyle\lim_{x\to 5^-} e^{\frac{x}{x-5}}$ (w) $\displaystyle\lim_{x\to\infty} \frac{e^{4x}-e^{2x}}{e^{-x}-e^{4x}}$ (x) $\displaystyle\lim_{x\to\pi/2^-} e^{\tan(x)}$

(y)) $\displaystyle\lim_{x\to\infty} 2^x$ (z) $\displaystyle\lim_{x\to\infty} 2^{-x}$ (a1) $\displaystyle\lim_{x\to\infty} \left(\frac{1}{2}\right)^x$

(b1) $\displaystyle\lim_{x\to\infty} \frac{x}{e^{-x}}$ (c1) $\displaystyle\lim_{x\to\infty} x\ln(x)$ (d1) $\displaystyle\lim_{x\to 0^+} (\sin(x))^{1/x}$

(e1) $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{e^x}$ (f1) $\displaystyle\lim_{x\to\pi/2^-} \frac{\tan(x)}{x-\pi/2}$ (g1) $\displaystyle\lim_{x\to\infty} \sqrt{x}-\sqrt{x-1}$

(h1) $\displaystyle\lim_{x\to-\infty} xe^{-x}$ (i1) $\displaystyle\lim_{x\to 0} \frac{1-e^{-x}}{\ln(x+1)}$ (j1) $\displaystyle\lim_{x\to 0} x^{\sin(x)}$ (k1) $\displaystyle\lim_{x\to\infty} (\ln(x))^{1/x}$

3. Suppose that the function $f(x)$ is given by: $f(x) = \begin{cases} x^2-1 & \text{if } x<2; \\ 0 & \text{if } x=2; \\ x-1 & \text{if } x>2. \end{cases}$

Compute the following limit, if it exists. If it does not exist, state why. $\displaystyle\lim_{x\to 2} f(x) =$

4. Let $f(x) = \begin{cases} \dfrac{x^2-16}{x-4} & \text{if } x \neq 4; \\ 8 & \text{if } x = 4. \end{cases}$

(a) Is f continuous at $x = 4$? Why or why not?

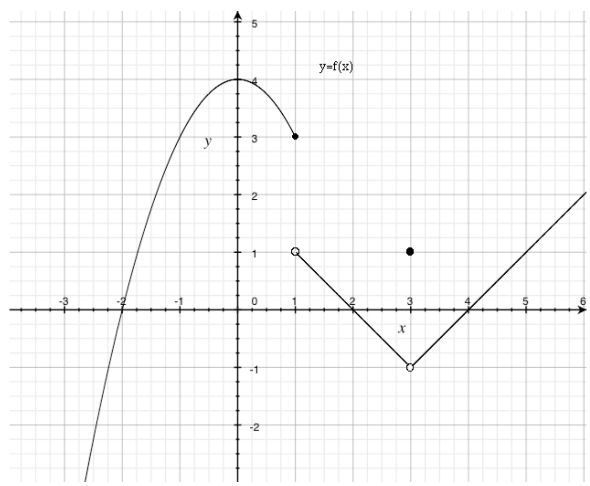
(b) Is f continuous on $(-\infty, 4)\cup(4,\infty)$? Why or why not?

5. Consider the graph at right of the function $F(x)$.
Find the following:

a) $\lim\limits_{x\to 2^-} F(x) =$ b) $\lim\limits_{x\to 2^+} F(x) =$

c) $\lim\limits_{x\to 2} F(x) =$ d) $F(2) =$

e) $\lim\limits_{x\to 0^-} F(x) =$ (f) $\lim\limits_{x\to 0} F(x) =$



6. (a) Where is the function $f(x) = \sqrt{25 - x^2}$ continuous?

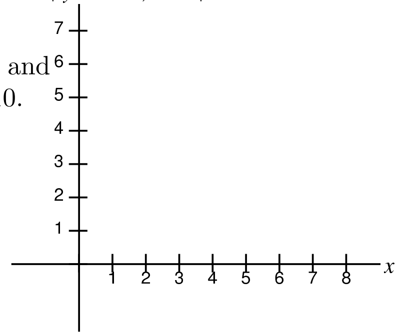
(b) Where is the function $g(x) = \dfrac{\sin(x)}{x^2 - x - 2}$ continuous?

7. Explain why $f(x) = e^{(x^2 - x)}$ is continuous on $(-\infty, \infty)$ and then use continuity to evaluate the following limit: $\lim\limits_{x\to 1} e^{(x^2-x)}$.
[i.e. If a function is continuous at $x = a$, what is $\lim\limits_{x\to a} f(x)$. Explain.

9. Write the $\delta - \epsilon$ definition of the following statement: $\lim\limits_{x\to 2} 5x - 7 = 3$.

10. The $\delta - \epsilon$ definition of the following statement: $\lim\limits_{x\to 3} 4x - 7 = 5$ is
For each $\epsilon > 0$, *there is a* $\delta > 0$ *such that if* $0 < |x - 3| < \delta$, *then* $|(4x - 7) - 5| < \epsilon$.
Prove this statement.

11. Graph the function $f(x) = 4x - 7$ on the graph at right, and then interpret the $\delta - \epsilon$ definition of the limit from question 10.



14. State in your own words what it means to say that $\lim\limits_{x\to 3} 4x - 7 = 5$.

15. Be able to answer questions regarding the origins of the derivative. Examples

Compute: $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ for $f(x) = \sqrt{x}$,
$f(x) = 3x - 2x^2 - 1$, and $f(x) = \frac{x}{x-1}$

16. Determine which is the graph of f and which is the graph of f'. Sketch the graph of f''.