## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

5. Exponents

Simplify and write the answer so all exponents are positive:

1. (M) $\left(u^{3} v^{5}\right)^{2} u^{4} v^{3}=$
2. (M) $\left(x^{-3} y^{7}\right)^{2}\left(5 x^{3} y^{-2}\right)^{2}=$
3. (KD) $\frac{\left(6 u^{3} v^{-5}\right)^{2}}{\left(9 u^{2} v^{-4}\right)^{3}}=$
4. (KD) $\frac{\left(2 a^{2} b^{-4}\right)^{3}}{\left(6 a^{-2} b^{3}\right)^{2}}=$
5. $(\mathrm{KD}) \frac{(2 x)^{2}\left(x^{3} y\right)^{4}}{x^{5}(4 y)^{3}}=$
6. (D) $\frac{4\left(x^{-2} y^{-4}\right)^{3}(3 y)^{2}}{\left(6 x^{-3} y^{4}\right)^{2}}=$

Essential Math Skills - Review Problems<br>MAT 250 - D. Ivanšić

### 6.2 Multiplying Polynomials

Simplify and write in standard form:

1. (M) $\left(3 x^{2}+4\right)(x-2)-x^{2}(2 x-4)=$
2. (M) $3 x\left(x^{2}-3\right)-(4 x-5)(x+3)=$
3. $(\mathrm{KD})(2 x+1)(3 x-7)+(2 x-7)(5 x+4)=$
4. $(\mathrm{KD})(4 x-3)\left(x^{2}+1\right)-(2 x+3)(x-7)=$
5. (KD) $(2 x-3)\left(x^{2}+9 x-8\right)=$
6. (D) $(x-2)\left(2 x^{2}-3 x-1\right)-\left(x^{2}+4\right)\left(x^{2}-3 x\right)=$

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

### 7.3 Factor Trinomials

Factor the expressions:

1. (E) $x^{2}-4 x-32=$
2. (E) $x^{2}-10 x-24=$
3. (M) $3 x^{2}+4 x-7=$
4. (M) $2 x^{2}+7 x-15=$
5. (KD) $6 x^{2}-29 x-5=$
6. (KD) $18 x^{2}-24 x+10=$
7. (KD) $12 x^{3}+x^{2}-6 x=$
8. (D) $4 x^{2}+27 x+45=$

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

### 7.4 Special Products and Factors

Expand the expressions:

1. (E) $(4 a-7)(4 a+7)=$
2. (E) $(9 x-\sqrt{2})(9 x+\sqrt{2})=$
3. (M) $\left(2 x^{2}-3 y\right)\left(2 x^{2}+3 y\right)$
4. (KD) $\left(3 x^{2} y^{2}-a b^{3}\right)\left(3 x^{2} y^{2}+a b^{3}\right)$
5. $(\mathrm{KD})(3 \sqrt{x}-4 \sqrt{y})(3 \sqrt{x}+4 \sqrt{y})$
6. (D) $\left(x \sqrt[3]{x}-y^{2} \sqrt[3]{y}\right)\left(x \sqrt[3]{x}+y^{2} \sqrt[3]{y}\right)$

Factor the expressions:
7. (E) $4 u^{2}-25 v^{2}=$
8. (E) $u^{3}-216=$
9. (M) $x^{6}-36 y^{8}=$
10. (M) $x^{6}+64 y^{9}=$
11. (KD) $50 x^{2} y^{6}-32 y^{2}=$
12. (KD) $32 x^{6}-108 y^{3}=$
13. (KD) $x^{\frac{2}{3}}-y^{\frac{2}{3}}=$
14. $(\mathrm{KD}) x^{\frac{3}{4}}+y^{\frac{3}{4}}=$

Essential Math Skills - Review Problems
MAT 250 - D. Ivanšić
8.3 Adding, Subtracting Rational Expressions

Simplify:

1. (M) $\frac{3 x+1}{x^{2}-25}-\frac{x-4}{x+5}=$
2. (M) $\frac{x+2}{x^{2}-2 x-3}-\frac{5 x}{x+1}=$
3. (KD) $\frac{2 x-3}{x^{2}+2 x-24}-\frac{x-7}{x^{2}+4 x-12}=$
4. (KD) $\frac{x}{2 x^{2}-3 x-35}+\frac{x-2}{x^{2}-6 x+5}=$
5. (KD) $\frac{3 x+7}{2 x^{2}+9 x-5}-\frac{2 x+1}{x^{2}+x-20}=$
6. (D) $\frac{2 x-5}{x^{2}-7 x+12}-\frac{2 x^{2}-4 x+7}{x^{3}-64}=$

Essential Math Skills - Review Problems
MAT 250 - D. Ivanšić

### 10.3 Composition of Functions

Find the composites $(f \circ g)(x)$ and $(g \circ f)(x)$ of the following functions.

1. (M) $f(x)=x-3 \quad g(x)=x^{2}-7 x+11$
2. (KD) $f(x)=\frac{3 x+2}{x-7} \quad g(x)=\frac{5 x}{2 x+1}$
3. (KD) $f(x)=\sqrt{x-3} \quad g(x)=\frac{2 x+1}{3 x^{2}-16}$

Consider the function $h(x)$ below and find two different solutions to the following problem: find functions $f$ and $g$ so that $h(x)=f(g(x))$, where neither $f$ nor $g$ are the identity function.
4. (M) $h(x)=\sqrt{7 x-1}$
5. $(\mathrm{M}) \quad h(x)=\frac{6}{x+4}$
6. $(\mathrm{KD}) h(x)=\frac{x^{2}+4}{x^{2}+7}$
7. (D) $h(x)=\frac{4}{x+\sqrt{x}}$

## Essential Math Skills - Review Problems

MAT 250 - D. Ivanšić

### 10.4 Graphing Functions

Graph the piecewise-defined functions below:

1. (KD) $f(x)= \begin{cases}3 x-2, & \text { if }-1<x<2 \\ 2 x-3, & \text { if } x \geq 2\end{cases}$
2. (KD) $f(x)= \begin{cases}-x-2, & \text { if }-3 \leq x<1 \\ -2 x+4, & \text { if } 1 \leq x<5\end{cases}$
3. (KD) $f(x)= \begin{cases}2 x-7, & \text { if } x \leq 1 \\ x+1, & \text { if } x>1\end{cases}$

A quadratic function is given below. Use the points found below to get a good graph of it without using the calculator.
a) Find the $x$-intercepts of its graph, if any. Find the $y$-intercept.
b) Find the vertex $(h, k)$ of the graph. Recall that $h=-\frac{b}{2 a}$.
4. (M) $f(x)=\frac{1}{2} x^{2}+4 x+6$
5. (M) $f(x)=-x^{2}-x+6$
6. (M) $f(x)=x^{2}-6 x+13$

## Essential Math Skills - Review Problems MAT 250 - D. Ivanšić

Evaluate without using the calculator.

1. (M) $\log _{9} 81=$
2. (M) $\log _{8} 256=$
3. (M) $\log _{3} 729=$
4. (KD) $\log _{7} \frac{1}{49}=$
5. (KD) $\log _{5} \frac{1}{125}=$
6. (KD) $\log _{6} \frac{1}{216}=$
7. (KD) $\log _{9} 3=$
8. (KD) $\log _{16} 2=$
9. (KD) $\log _{100} 1000=$
10. (KD) $\log _{c} \sqrt[4]{c^{3}}=$
11. (KD) $\log _{a^{3}} a^{12}=$
12. (D) $\log _{c^{2}} \sqrt[5]{c^{2}}=$

Simplify the following expressions.
13. (M) $\left(e^{x}+e^{-x}\right)^{2}=$
14. (M) $\left(2^{x}-2^{-x}\right)\left(2^{x}+2^{-x}\right)=$
15. (M) $\left(e^{x}-1\right)\left(e^{2 x}+e^{x}+1\right)=$
16. (M) $e^{\ln (x-1)}=$
17. (KD) $\frac{e^{2 x}+4 e^{x}+4}{e^{x}+2}=$
18. (KD) $e^{2 \ln (x+3)}=$
19. $(\mathrm{KD}) e^{\frac{\ln x}{3}}=$
20. (KD) $\ln \sqrt{e^{-3 x}}=$
21. (D) $e^{\frac{\ln x}{2}}\left(e^{3 \ln x}\right)^{2}=$

Write as a sum or difference of logarithms. Express powers as factors. Simplify if possible.
22. (M) $\ln \left(e^{3} x^{8} y^{-2}\right)=$ 23. (M) $\log _{6}\left(216\left(x^{2}+5 x-24\right)^{3}\right)=$ 24. (M) $\log _{3}\left(81 x^{4} y^{2}\right)=$
25. (M) $\ln \frac{x^{3} y^{4}}{x^{-2} y^{8}}=\quad$ 26. (KD) $\log _{8} \frac{x^{-\frac{8}{3}} y^{2}}{64 \sqrt[3]{x} y^{4}}=$ 27. (KD) $\log _{5} \sqrt[5]{\frac{x^{3} y^{-7}}{25 y^{3}}}=$

Write as a single logarithm. Simplify if possible.
28. (KD) $2 \log \left(5 x^{3}\right)+\frac{1}{2} \log \left(625 y^{4}\right)-5 \log x=$
29. (KD) $\frac{1}{2} \log \left(49 x^{\frac{3}{4}}\right)+\frac{1}{4} \log \left(16 x^{7}\right)=$
30. (KD) $\frac{1}{3} \ln \left(125 x^{\frac{1}{2}}\right)+\frac{1}{2} \ln \left(625 y^{7}\right)-\ln \left(x^{\frac{2}{3}}\right)=$
31. (KD) $\log _{2}(x-4)+3 \log _{2}(x+4)-2 \log _{2}\left(x^{2}-16\right)=$
32. $(\mathrm{KD}) 3 \log _{b}\left(x^{2}+4 x-21\right)-2 \log _{b}(x+7)-5 \log _{b}(x-3)=$
33. $(\mathrm{KD}) 2 \log _{a}(x+6)-4 \log _{a}\left(x^{2}-36\right)-3 \log _{a}(x-6)=$

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

12. Solving Equations

Solve the equations.

1. (M) $2 x e^{x}+x^{2} e^{x}=0$
2. (M) $2+\frac{2 x+1}{x+4}=\frac{x-3}{x+4}$
3. (M) $3(x+2)^{2}(x-7)^{4}+4(x+2)^{3}(x-7)^{3}=0$
4. (M) $4(2 x-1)(x-3)^{4}-4(2 x-1)^{2}(x-3)^{3}=0$
5. (KD) $4 x^{4}-11 x^{2}-3=0$
6. (KD) $\frac{x+3}{x+1}+\frac{8 x}{x^{2}-6 x-7}=\frac{7}{x-7}$
7. $(\mathrm{KD}) \frac{x}{x-1}-\frac{2 x+30}{x^{2}+2 x-3}=\frac{6}{x+3}$
8. (KD) $x+\sqrt{40-3 x}=4$
9. $(\mathrm{KD}) 2 x+4=x-\sqrt{6 x+51}$
10. (KD) $(2 x-7) \sqrt{x+2}-\frac{x^{2}-7 x+1}{2 \sqrt{x+2}}=0$
11. (KD) $2 x\left(x^{2}+1\right)^{\frac{2}{3}}+\frac{4}{3} x^{3}\left(x^{2}+1\right)^{-\frac{1}{3}}=0$
12. $(\mathrm{KD}) \log _{2}(x+1)+\log _{2}(x+5)=5$
13. $(\mathrm{KD}) \log _{3}(x+14)+\log _{3}(x-10)=4$
14. (D) $\sqrt{4 x+5}+\sqrt{x+5}=3$

## Essential Math Skills - Review Problems MAT 250 - D. Ivanšić

## 13. Solving Inequalities

Solve the inequalities and write the solution in interval form.

1. (E) $4 \leq 3 x-1<21$
2. (E) $8 \leq 3 x-1<11$
3. (M) $|x-7|>3$
4. (M) $|2 x-6|<9$
5. (M) $|x+5| \leq 9$
6. (M) $|2 x-3| \geq 7$

Use the graph of a quadratic function to help you solve the following inequalities
7. (KD) $x^{2}-5 x \geq 0$
8. (KD) $x^{2}-4 x<5 x-20$
9. (KD) $-2 x^{2}+x-3 \leq 0$
10. (KD) $\frac{x-7}{2 x+3}>0$
11. (D) $\left|x^{2}-8\right| \leq 2$
12. (D) $\left|x^{2}+2 x-15\right|>7$

## Essential Math Skills - Review Problems MAT 250 - D. Ivanšić

## 14. Angles

1. (M) Indicate both the radian and degree measure under the following angles. (Use equally-spaced lines to help you determine what the angles are.)

2. (M) Sketch angles in standard position with indicated radian measure.
$\frac{7 \pi}{6}$
$\frac{3 \pi}{7}$
$-\frac{4 \pi}{3}$
$\frac{34 \pi}{5}$
$-\frac{13 \pi}{8}$

## Essential Math Skills - Review Text <br> MAT 250 - D. Ivanšić

## 15. Trig. Functions of Standard Angles

We can evaluate trigonometric functions at multiples of $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$ with little memorizing and by simply reading the picture. (Note that the discussion that follows applies to "noneasy" multiples of those angles, whose terminal side is not on the $x$ - or $y$-axis; for example, for angle $3 \frac{\pi}{3}=\pi$, trigonometric function values are obvious, because its associated point on the unit circle is obviously $(-1,0)$ ).

First, we need to memorize these six numbers and their relative sizes:
values of cosines and sines of standard angles are $\frac{1}{2}<\frac{\sqrt{2}}{2}<\frac{\sqrt{3}}{2} \quad(\approx 0.5<0.7<0.9)$
values of tangents and cotangents of standard angles are $\frac{1}{\sqrt{3}}<1<\sqrt{3} \quad(\approx 0.6<1<1.7)$
That $\frac{\sqrt{2}}{2}$ is associated with multiples of $\frac{\pi}{4}$ simply has to be memorized, so

$$
\cos \frac{\pi}{4}=\frac{\sqrt{2}}{2} \quad \sin \frac{\pi}{4}=\frac{\sqrt{2}}{2}
$$

The task is to figure out which of the values $\frac{1}{2}, \frac{\sqrt{3}}{2}$ cosines and sines take for multiples of $\frac{\pi}{6}$ and $\frac{\pi}{3}$.


Consider the angles $\frac{\pi}{6}$ and $\frac{\pi}{3}$. When drawing the picture, try to be reasonably accurate in placement of the angle, but the most important feature of the picture is that
the terminal side of $\frac{\pi}{6}$ is "closer" to the $x$-axis the terminal side of $\frac{\pi}{3}$ is "closer" to the $y$-axis

From this we infer that the lengths of sides of the indicated triangles have the following relationship:

$$
a>b \text { and } c<d
$$

Because $a, b, c$ and $d$ can only be $\frac{1}{2}$ or $\frac{\sqrt{3}}{2}$, we conclude that

$$
a=\frac{\sqrt{3}}{2} \quad b=\frac{1}{2} \quad c=\frac{1}{2} \quad d=\frac{\sqrt{3}}{2}
$$

and therefore, as $a, b, c$ and $d$ are the $x$ - and $y$-coordinates of points on the unit circle associated to angles $\frac{\pi}{6}$ and $\frac{\pi}{3}$, and therefore their cosines and sines,

$$
\cos \frac{\pi}{6}=\frac{\sqrt{3}}{2} \quad \sin \frac{\pi}{6}=\frac{1}{2} \quad \cos \frac{\pi}{3}=\frac{1}{2} \quad \sin \frac{\pi}{3}=\frac{\sqrt{3}}{2}
$$

The tangent of an angle can be read on the line $x=1$ : it is the $y$-coordinate of the intersection of the terminal side of the angle (or its extension past the origin) with the line $x=1$.

The terminal side of $\frac{\pi}{4}$ is midway between the $x$ - and $y$-axes, so by symmetry it will intersect the line $x=1$ at the point $(1,1)$. From the fact that the terminal side of $\frac{\pi}{6}$ is "closer" to the $x$-axis and that the terminal side of $\frac{\pi}{3}$ is "closer" to the $y$-axis it is clear that $e<1$ and $f>1$. Because $e$ and $f$ can only be $\frac{1}{\sqrt{3}}$ or $\sqrt{3}$, we conclude $e=\frac{1}{\sqrt{3}}$ and $f=\sqrt{3}$. Therefore,

$$
\tan \frac{\pi}{6}=\frac{1}{\sqrt{3}} \quad \tan \frac{\pi}{4}=1 \quad \tan \frac{\pi}{3}=\sqrt{3}
$$

For general multiples of $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$ we just need to draw a picture and infer trigonometric function values from it.

Example. Determine the values of trigonometric functions of angles $\frac{5 \pi}{6}, \frac{7 \pi}{4}$ and $-\frac{2 \pi}{3}$.




Angle $\frac{5 \pi}{6}$. The terminal side of this angle is clearly "closer" to the $x$-axis, so for the sides of the appropriate triangle we conclude $a>b$, thus $a=\frac{\sqrt{3}}{2}$ and $b=\frac{1}{2}$. Lengths $a$ and $b$ are equal to sizes of the $x$ - and $y$-coordinates, and the position of the point $P$ determines the signs of those coordinates. For angle $\frac{5 \pi}{6}$, the point $P$ has a negative $x$ - and positive $y$-coordinate, so we conclude:

$$
\cos \frac{5 \pi}{6}=-\frac{\sqrt{3}}{2} \quad \sin \frac{5 \pi}{6}=\frac{1}{2}
$$

To read the tangent of the angle on the line $x=1$, we extend the terminal side past the origin and look where this line crosses the line $x=1$. From the picture, since the line is "closer" to the $x$-axis, we infer $c<1$, so $c=\frac{1}{\sqrt{3}}$. Since tangent is the $y$-coordinate of the intersection, which is below the $x$-axis, we get $\tan \frac{5 \pi}{6}=-\frac{1}{\sqrt{3}}$.
Other values of trigonometric functions are simply computed from the values of cosine, sine and tangent using basic trigonometric identities:

$$
\sec \frac{5 \pi}{6}=\frac{1}{\cos \frac{5 \pi}{6}}=-\frac{2}{\sqrt{3}}, \quad \csc \frac{5 \pi}{6}=\frac{1}{\sin \frac{5 \pi}{6}}=-2, \quad \cot \frac{5 \pi}{6}=\frac{1}{\tan \frac{5 \pi}{6}}=-\sqrt{3}
$$

Angle $\frac{7 \pi}{4}$. The terminal side of this angle is clearly "equidistant" to the $x$ and $y$-axes, so for the sides of the appropriate triangle we conclude $a=b$. Because this is a multiple of $\frac{\pi}{4}$, we must have $a=\frac{\sqrt{2}}{2}$ and $b=\frac{\sqrt{2}}{2}$. For angle $\frac{7 \pi}{4}$, the point $P$ has a positive $x$ - and negative $y$-coordinates, so we conclude:

$$
\cos \frac{7 \pi}{4}=\frac{\sqrt{2}}{2} \quad \sin \frac{7 \pi}{4}=-\frac{\sqrt{2}}{2}
$$

To read the tangent of the angle on the line $x=1$, we look where the terminal side crosses the line $x=1$. From the picture, since the line is "equidistant" to the $x$ and $y$-axes, we infer $c=1$. Since tangent is the $y$-coordinate of the intersection, which is below the $x$ axis, we get $\tan \frac{7 \pi}{4}=-1$. Other trigonometric function values are $\sec \frac{7 \pi}{4}=\frac{2}{\sqrt{2}}=\sqrt{2}$, $\csc \frac{7 \pi}{4}=-\frac{2}{\sqrt{2}}=-\sqrt{2}, \cot \frac{7 \pi}{4}=-1$.

Angle $-\frac{2 \pi}{3}$. The terminal side of this angle is clearly "closer" to the $y$-axis, so for the sides of the appropriate triangle we conclude $a<b$, thus $a=\frac{1}{2}$ and $b=\frac{\sqrt{3}}{2}$. For angle $-\frac{2 \pi}{3}$, the point $P$ has negative $x$ - and $y$-coordinates, so we conclude:

$$
\cos \left(-\frac{2 \pi}{3}\right)=-\frac{1}{2} \quad \sin \left(-\frac{2 \pi}{3}\right)=-\frac{\sqrt{3}}{2}
$$

To read the tangent of the angle on the line $x=1$, we extend the terminal side past the origin and look where this line crosses the line $x=1$. From the picture, since the line is "closer" to the $y$-axis, we infer $c>1$, so $c=\sqrt{3}$. Since tangent is the $y$-coordinate of the intersection, which is above the $x$-axis, we get $\tan \left(-\frac{2 \pi}{3}\right)=\sqrt{3}$. Other trigonometric function values are $\sec \left(-\frac{2 \pi}{3}\right)=-2, \csc \left(-\frac{2 \pi}{3}\right)=-\frac{2}{\sqrt{3}}, \cot \left(-\frac{2 \pi}{3}\right)=\frac{1}{\sqrt{3}}$.

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

## 15. Trigonometric

## Function Values

Use the unit circle and the line $x=1$ to estimate the values of the trigonometric functions of the angles drawn.


1. (M) $\cos \alpha=\quad \tan \alpha=\quad \csc \alpha=$
2. (M) $\sin \beta=\quad \cot \beta=\quad \sec \beta=$
3. (M) $\sin \gamma=\quad \tan \gamma=\quad \sec \gamma=$
4. (M) $\cos \delta=\quad \cot \delta=\quad \csc \delta=$

For each of the following, draw the unit circle and the appropriate angle in order to infer from the picture the exact values of the trigonometric functions.
5. (M) $\sin 210^{\circ}=$ $\tan \frac{2 \pi}{3}=$
$\csc \left(-90^{\circ}\right)=\quad \cot \left(-\frac{\pi}{4}\right)=$
6. (M) $\cot \left(-150^{\circ}\right)=$
$\sec \left(-\frac{5 \pi}{6}\right)=$
$\cos \frac{5 \pi}{3}=$
$\csc 225^{\circ}=$
7. (M) $\sec 495^{\circ}=$ $\tan \left(-\frac{9 \pi}{2}\right)=$ $\cos \left(-600^{\circ}\right)=\quad \sin \frac{47 \pi}{3}=$
8. (M) If $\cos \theta=-\frac{3}{7}$ and $\theta$ is in the third quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.
9. (M) If $\tan \theta=4$ and $\theta$ is in the third quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.
10. (M) If $\sin \theta=\frac{3}{8}$ and $\theta$ is in the second quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.
11. (M) If $\sec \theta=\frac{15}{2}$ and $\theta$ is in the fourth quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.
12. (M) If $\cot \theta=-\frac{1}{3}$ and $\theta$ is in the second quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.
13. (M) If $\csc \theta=-7$ and $\theta$ is in the fourth quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

## 18. Trigonometric Identities

Show the identity.

1. (M) $\sin \theta \csc \theta-\cos ^{2} \theta=\sin ^{2} \theta$
2. (M) $\tan \theta(\sec \theta+\tan \theta)=\sec \theta(\sec \theta+\tan \theta)-1$
3. (M) $\frac{\sin (\alpha+\beta)}{\cos \alpha \cos \beta}=\tan \alpha+\tan \beta$
4. (M) $1-\frac{\sin ^{2} \theta}{1+\cos \theta}=\cos \theta$
5. $(\mathrm{KD}) \frac{\sin \theta \cos \theta}{(\cos \theta-\sin \theta)(\cos \theta+\sin \theta)}=\frac{1}{2} \tan (2 \theta)$
6. (KD) $\frac{1}{1-\sin \theta}+\frac{1}{1+\sin \theta}=2 \sec ^{2} \theta$
7. (D) $\frac{\cos \theta}{1+\sin \theta}+\frac{1+\sin \theta}{\cos \theta}=2 \sec \theta$
8. (KD) Develop the formula for $\cos (4 \theta)$ by using double-angle identities. The final expression should only have $\sin \theta$ and $\cos \theta$ in it.
9. (KD) Develop the formula for $\cos (3 \theta)$ by starting as follows and using sum and doubleangle identities. The final expression should only have $\sin \theta$ and $\cos \theta$ in it.
$\cos (3 \theta)=\cos (2 \theta+\theta)=$

## Essential Math Skills - Review Problems MAT 250 - D. Ivanšić

## 19. Inverse

Trigonometric Functions
Use the unit circle and the line $x=1$ to draw the angles requested. Some may be undefined.


1. (M) $\arccos 0.25 \quad \arcsin 0.9 \quad \arctan 0.3$
2. (M) $\arctan (-1.2)$
$\arccos (-0.55)$
$\arcsin (-0.15)$
3. (M) $\arccos (-1.4)$
$\arctan 2$
$\arcsin (-1)$

For each of the following, use the unit circle and the line $x=1$ to help you find the exact angles of the inverse trigonometric functions. Some may be undefined.
4. (M) $\arcsin \frac{\sqrt{3}}{2}=\quad \arccos \frac{\sqrt{2}}{2}=\quad \arctan \frac{1}{\sqrt{3}}=\quad \arctan \sqrt{3}=$
5. (M) $\arccos \left(-\frac{1}{2}\right)=\quad \arctan (-1)=\quad \arcsin \left(-\frac{\sqrt{2}}{2}\right)=\quad \arctan \left(-\frac{1}{\sqrt{3}}\right)=$
6. (M) $\arcsin \sqrt{3}=$ $\arcsin \left(-\frac{1}{2}\right)=\quad \arctan 1=\quad \arccos \frac{2}{\sqrt{3}}=$

Find the exact value of the expressions. For some of them, you will need a picture.
7. $(\mathrm{M}) \sin (\arcsin 0.83)=$
8. (KD) $\arccos \left(\cos \frac{2 \pi}{7}\right)=$
9. $(\mathrm{KD}) \arctan \left(\tan \left(-\frac{4 \pi}{9}\right)\right)=$
10. (D) $\tan \left(\arccos \left(-\frac{1}{3}\right)\right)=$
11. (D) $\sin \left(2 \arccos \left(-\frac{4}{7}\right)\right)=$
$\tan (\arctan 5)=$
$\arcsin \left(\sin \frac{7 \pi}{5}\right)=$
$\arctan \left(\tan \frac{5 \pi}{9}\right)=$
$\sin (\arctan (-5))=$
$\tan \left(2 \arcsin \left(-\frac{7}{8}\right)\right)=$

## Essential Math Skills - Review Problems <br> MAT 250 - D. Ivanšić

## 20. Trigonometric <br> Equations

Use the unit circle and the line $x=1$ to draw all the angles in $[0,2 \pi)$ that satisfy the equation. Some equations may have no solutions.


1. (M) $\sin \theta=0.25$
$\cos \theta=0.8$
$\tan \theta=0.4$
2. (M) $\cot \theta=-1.2$
$\cos \theta=-0.15$
$\sec \theta=1.5$
3. (M) $\sin \theta=7$
$\csc \theta=2.4$
$\cos \theta=-1$

Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0,2 \pi)$.
4. (M) $2 \sin \theta-\sqrt{2}=0$
5. (KD) $2 \cos (2 \theta)-\sqrt{3}=0$
6. (KD) $2 \cos ^{2} \theta+\cos \theta-1=0$
7. (KD) $2 \cos ^{2} \theta-5 \cos \theta-3=0$
8. (KD) $\sin (2 \theta)+2 \sin ^{2} \theta=0$
9. (KD) $\sec ^{2} \theta=6 \tan \theta+8$

