1.0 Prologue

What is this course about?

Example. How can you tell when a natural number is divisible by 4?

1.1 Statements and Conditional Statements

A *statement* (or *proposition*) is a sentence that is true or false (but not both). In particular, its truthfulness can be determined.

Example. Which of the following are statements? Are they true or false?

$3 + 5 = 10$

$5 \cdot 11 = 55$

$x + 4 = 7$

For $x = -3$, $x + 4 = 7$.

$x + 4 = 7$ for only one real number x.

The U.S. debt is huge.

Is there a number whose cube is 7?

If x is divisible by 10, then it is divisible by 5.

$(a + b)^2 = a^2 + 2ab + b^2$

$(a + b)^2 = a^2 + 2ab + b^2$ for all real numbers a, b.

$(a + b)^4 = a^4 + 4ab + b^4$ for all real numbers a, b.

If a triangle is isosceles, it has two angles of equal measure.

How to determine if a statement is true or false?

Previous knowledge

Examples (or counterexamples)

Cooperation

Conditional Statements. Statements often come in conditional form, "If P, then Q," or $P \implies Q$, where P and Q are sentences.

P is called the *hypothesis* and Q is called the *conclusion*.

Truthfulness of $P \implies Q$ should depend on the truthfulness of P and Q.

Example. Consider the conditional statement:

If you jump over the fence, then I give you an apple.

Consider the whether you would say this statement is false under the following possibilities

You jump over the fence	I give you an apple	
You jump over the fence	I don't give you an apple	
You don't jump over the fence	I give you an apple	
You don't jump over the fence	I don't give you an apple	

Example. Consider the conditional statement about a real number x:

$$\text{If } x > 3, \text{ then } x^2 - 4x + 3 > 0.$$

Is there an x for which the statement is false?

x	Hypothesis	Conclusion	Truthfulness of statement for particular x
$x = 4$			
$x = 7$			
$x = 0$			
$x = 2$			

To show statement is false for some x, we would need to find an x so that $x > 3$ and $x^2 - 4x + 3 \leq 0$.

Note: not being able to find such a number doesn't mean the statement is true for every x, we merely haven't been able to show it false for some x.

To show the statement is true for every x, we can do several approaches:

a) Graph

b) Complete the square

c) Factor

The previous two examples suggest the following dependence of truthfulness of $P \implies Q$ on the truthfulness of P and Q (truth table):

P	Q	$P \implies Q$
T	T	
T	F	
F	T	
F	F	

Notes:

Example. Based on table, the following statements are true:

If $7 < 1$, then $2 + 3 = 5$

If $7 < 1$, then $2 + 3 = 10$

Now that we know that *If* $x > 3$*, then* $x^2 - 4x + 3 > 0$ is true for every real x, we can say:

$5 > 3$, so

$1.5 < 3$, so

$3.1 > 3$, so

Closure properties of number systems under operations

The following letters are standard designations for the given sets of numbers:

natural numbers: **N** integers: **Z** rational numbers: **Q** real numbers: **R**

The set of natural numbers **N** is closed under addition and multiplication, not under subtraction nor division.

1.2 Constructing Direct Proofs

A *definition* is an agreement that a word or a phrase will stand for some object, property or concept.

Definition. An integer a is *even*, if there exists an integer n such that $a = 2n$.
An integer a is *odd*, if there exists an integer n such that $a = 2n + 1$.

Proposition. If x is and even integer, and y is an integer, then $x \cdot y$ is an even integer.

Proof. To try to prove the proposition, we can use a "know-show" table, which is a way to link the hypothesis and the conclusion.

Know	Why
x even, y integer	hypothesis
$x \cdot y$ is even	conclusion
Show	

Write the proof in words.

In the book, read the guidelines for constructing a direct proof — don't have to follow them verbatim.