

College Algebra — Joysheet 1  
MAT 140, Spring 2023 — D. Ivanšić

Name: \_\_\_\_\_

Covers: JIT 6, 7, 12–14, 17      Show all  
your work!

Write interval notation and sketch on the number line.

1. (3pts)  $\{x \mid -2 \leq x < 4\}$

2. (3pts)  $\{x \mid x \leq 5\}$

Solve the equations.

3. (3pts)  $2x - 6 = -3x + 4$

4. (4pts)  $5(z - 3) - 1 = z + 4(2z + 3)$

Simplify and write in standard form:

5. (4pts)  $(x + 2)^3 - 4x^2 + x =$

6. (4pts)  $x(2x + 3) - (x - 7)(x + 3) =$

Simplify and write the answer so all exponents are positive:

7. (2pts)  $a^2(2b)^4 =$

8. (2pts)  $\frac{(3x)^2}{x^5} =$

9. (3pts)  $(u^4v^{-5})^2 u^{-4}v^3 =$

10. (5pts)  $(x^{-5}y^4)^{-2}(4x^{-3}y^5)^3 =$

11. (7pts)  $\frac{(6u^{-2}v^5)^3}{(8u^4v^{-3})^2} =$

Factor the following.

12. (4pts)  $x^2 - 5x - 14 =$

13. (4pts)  $x^2 + 8x - 48 =$

Solve the equations.

14. (6pts)  $x^2 + 3x = 2x + 20$

15. (6pts)  $2x^2 - 10x + 5 = x^2 - 16$

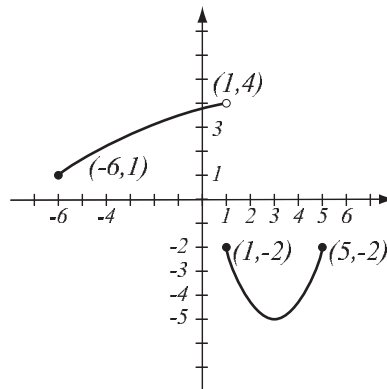
1. (11pts) Draw the triangle with vertices  $A = (-4, 1)$ ,  $B = (-2, 5)$  and  $C = (0, -1)$  in the coordinate plane.

- a) Does it look like the triangle is isosceles (has two sides of equal length)?
- b) By computing lengths of all sides, find out algebraically whether  $ABC$  is isosceles.

2. (10pts) Find the equation of the circle, if  $(-1, -2)$  and  $(1, 4)$  are at the ends of a diameter. Draw the circle.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- a) Find  $f(-6)$  and  $f(1)$ .
- b) What is the domain of  $f$ ?
- c) What is the range of  $f$ ?
- d) What are the solutions of the equation  $f(x) = -4$ ?



4. (12pts) The function  $f(x) = -x^2 + 8x + 5$  is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (9pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{x - 3}{2x + 6}$$

$$g(x) = \frac{5 + \sqrt{x}}{x - 2}$$

6. (10pts) Let  $h(x) = 3x^2 - 5x + 7$ . Find the following (simplify where appropriate).

$$h(1) =$$

$$h(-3) =$$

$$h(2\sqrt{u}) =$$

$$h(x - 3) =$$



5. (12pts) The water bill for a household was \$39.56 in a month when it used 40 hundred-gallons of water. In another month, it used 34 hundred-gallons and was billed \$37.04.
- Assuming that water cost  $C(x)$  is a linear function of the amount of water  $x$  used (in hundred-gallons), write a formula for  $C(x)$ .
  - What is the cost if no water is used during a month? What is the meaning of this number?
  - What is the meaning of the slope in this example?

6. (20pts) A statistician is trying to establish a trend for the population of Kentucky. In the table,  $P$  is the population, in millions, of Kentucky in year  $x$ . Solve the problems below with accuracy 6 decimal points.
- Draw the scatterplot of the data. Does the relationship look linear?
  - Use two points in the scatterplot to get an equation of a line that models the relationship between  $P$  and  $x$ . Draw the line on the graph.
  - Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.
  - Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $x$  and  $P$ ?
  - What population of Kentucky can we expect for 2027?

$x$	$P$
1970	3.2
1980	3.7
1990	3.7
2000	4.0
2010	4.3
2020	4.5

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $-5 \leq 4x - 1 < 15$

2. (7pts)  $3x - 2 < 8$  or  $5x + 4 > 29$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{2 - 5x}}{x + 1}$ .

4. (14pts) Lawyers Mitch and James charge for legal services as follows: Mitch charges \$150 per hour, and James charges \$300 for the first three hours and then \$170 per hour for the hours after three. Suppose your case requires at least three hours of legal services. For which number of hours does Mitch have have the better deal? Solve as an inequality.

**5.** (14pts) At the same time, Jayla started driving from Paducah, and Tiana started driving from Elizabethtown. They drove toward each other along the same road and met on this road after one hour and 30 minutes. The distance from Paducah to Elizabethtown is 171 miles and Jayla drove 5 mph faster than Tiana.

- a) How fast was each woman driving?
- b) How far from Paducah did the women meet?

**6.** (14pts) Amanda invested some money at 4% simple interest, and some at 5% simple interest. If her total interest over 1 year was \$97, and she invested \$500 more at 5% than she did at 4%, how much did she invest at each interest rate?



1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{9x}{x^2 + x + 10}$ . (When entering function into calculator, don't forget to put parentheses around the denominator.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

- a) Find the local maxima and minima for this function.
- b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = \frac{7}{x-3}$ ,  $g(x) = 4 + \sqrt{x}$ . Find the following (simplify where possible):

$(f - g)(1) =$   $(fg)(-4) =$

$\frac{f}{g}(x) =$   $(g \circ f)(4) =$

$(f \circ g)(x) =$

The domain of  $(fg)(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = \sqrt{5 - x^2}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

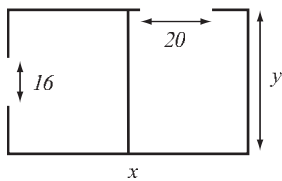
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x + 1, & \text{if } -2 < x \leq 3 \\ x - 5 & \text{if } x > 3. \end{cases}$$

5. (14pts) Using a fence, farmer Hilda wishes to enclose a rectangular field with area 20000 square feet, divide into parts and leave 16 ft and 20 ft openings in the fence, as in the picture. She wishes to minimize the total length of the fence.

a) Express the total length of the fence as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the field for which the total length of the fence is minimal? What is the minimal fence length?



College Algebra — Joysheet 6  
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Name: \_\_\_\_\_

Covers: 2.4, 2.5

Show all your work!

1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 4x^2 + 1$$

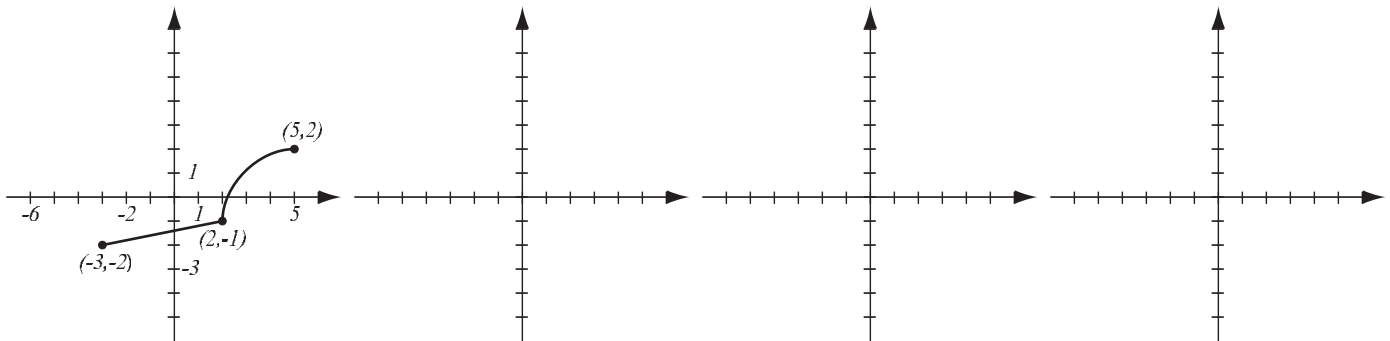
$$g(x) = -x^2 + 5|x| + 4$$

$$h(x) = x^3 - 15x$$

2. (16pts) Draw the graphs of  $f(x) = \frac{1}{x-4} + 3$  and  $g(x) = -3\sqrt{x} + 2$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = x^2$ , shifted right 2 units
  - shape of  $y = |x|$ , stretched vertically by factor 4, then reflected about the  $x$ -axis
  - shape of  $y = x^3$ , shifted left 2 units, then reflected about the  $y$ -axis, then stretched horizontally by factor 3.

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(-x)$ ,  $-f(x + 3)$  and  $f(2x) - 3$  and label all the relevant points.



College Algebra — Joysheet 7  
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Name: \_\_\_\_\_

*Covers: JIT 13, 15, 21-23 Show all your work!*

Use formulas to expand:

1. (4pts)  $(3x - y)^2 =$

2. (5pts)  $(u^2 + 4y)^2 =$

3. (4pts)  $(5x - 2y)(5x + 2y) =$

Use the *ac*-method or another method to factor. Show how you got your answer.

4. (5pts)  $3x^2 - 13x - 10 =$

5. (6pts)  $12x^2 + 8x - 15 =$

6. (8pts) Compute expressions with fractions by hand.

$$\frac{4}{7} \cdot \frac{35}{16} =$$

$$\frac{2}{3} \div \frac{14}{9} =$$

$$\frac{7}{5} + \frac{4}{15} =$$

$$\frac{9}{14} - \frac{11}{35} =$$

Multiply or divide the rational expressions.

7. (7pts)  $\frac{x+1}{x^2-16} \cdot \frac{3x-12}{2x^2+8x+6} =$

8. (7pts)  $\frac{4x+2}{3x+9} \div \frac{2x^2-x-1}{x^2+2x-3} =$

Add or subtract the rational expressions.

9. (6pts)  $\frac{x-1}{x-3} - \frac{8x-2}{x^2+5x-24} =$

10. (8pts)  $\frac{x+2}{4x^2+3x-7} + \frac{x-5}{x^2-10x+9} =$

College Algebra — Joysheet 8  
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*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $2i(-3 + 5i) - (5i)^2 =$

2. (6pts)  $\frac{2 + 3i}{2 - i} =$

3. (4pts) Simplify and justify your answer.

$i^{58} =$

4. (8pts) The number of plants for sale in a nursery is described by the function  $N(x) = x^2 - 16x + 100$ , where  $x$  is the number of days after April 22nd.

a) On what dates did the nursery have 52 plants for sale?

b) On what date did the number of plants for sale bottom out?

5. (8pts) Solve the equation:  $x^4 - 3x^2 - 12 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 12x + 53 = 0$

**7.** (12pts) The quadratic function  $f(x) = x^2 + 4x + 8$  is given. Do the following without using the calculator.

- a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

**8.** (12pts) We wish to alter a 5 ft  $\times$  11 ft plot in the garden by lengthening the 5 ft side and shortening the 11 ft side by the same amount. If the area of the new rectangular plot is to be 62 ft<sup>2</sup>, by how much do we change the lengths of the sides?



College Algebra — Joysheet 9  
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Name: \_\_\_\_\_

*Covers: 3.3, 3.4, 3.5 Show all your work!*

1. (4pts) Solve the equation.

$$|2x + 3| = 1$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x + 3| < 4$$

$$|2x - 1| > 5$$

Solve the equations:

3. (8pts)  $\frac{x - 7}{x + 2} + \frac{x + 5}{x - 2} = \frac{x^2 - 8x + 40}{x^2 - 4}$

4. (8pts)  $2x - 2 = x + \sqrt{39 - 2x}$

5. (14pts) An arrow is launched from height 10 meters upwards with initial velocity 40 meters per second. Its height in meters after  $t$  seconds is given by  $s(t) = -5t^2 + 40t + 10$ .

a) Sketch the graph of the height function.

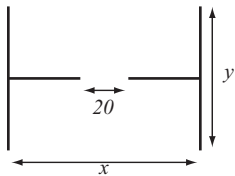
b) When does the arrow reach its greatest height, and what is that height?

c) When is the arrow at height 56.2 meters?

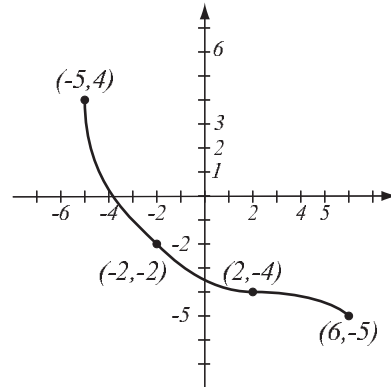
6. (14pts) A highway maintenance department is planning a building with two stalls intended to house its equipment. The stalls are connected by a 20ft wide door. The maintenance department has budgeted for total wall length 400 feet and its goal is to maximize the enclosed area.

a) Express the area of the building (the entire rectangle) as a function of one of the sides of the rectangle. What is the domain of this function?

c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the greatest area and what is the greatest area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $f(x) = \frac{2x}{x+3}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_5 125 =$$

$$\log \frac{1}{100} =$$

$$\log_{27} 3 =$$

$$\log_{a^3} a^{15} =$$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_7 0.7$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$1,000 and annual interest rate of 4.14%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	$n$	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of  $f(x) = \ln(7 - 2x)$ .

7. (8pts) The expression that converts temperatures in degrees Celsius (C) to degrees Fahrenheit (F) is  $F(C) = 32 + \frac{9}{5}C$ .

- Find temperatures in degrees Fahrenheit for  $-5^\circ\text{C}$ , and  $20^\circ\text{C}$ .
- Find a formula for the inverse function and explain what it represents.
- What temperatures in degrees Celsius correspond to  $41^\circ\text{F}$  and  $95^\circ\text{F}$ ?

8. (7pts) Using transformations, draw the graph of  $f(x) = 4 - e^x$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If  $\log_a 7 = 0.6868$  and  $\log_a 3 = 0.3878$ , calculate:

$$\log_a 21 = \qquad \qquad \qquad \log_a \frac{9}{7} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 (64x^3y^7) =$$

$$\log \frac{1000\sqrt[3]{x^5y^6}}{x^{\frac{2}{3}}z^3} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$3\log_5(2x^4) + 4\log_5 x^2 - 2\log_5(2y^3) =$$

$$\log_3(x^2 - 16) + 2\log_3(x + 4) - \log_3(x^2 - 7x + 12) =$$

4. (3pts) Simplify.  $\log_4 4^{2x-1} =$   $e^{\ln(3x)} =$

Solve the equations.

5. (5pts)  $7^{2x-3} = 49^{3x}$

6. (7pts)  $3^{x-1} = 4^{2x}$

7. (5pts) A tire shop bought a car jack for \$1,400. The value of the car jack each year is 85% of the value of the year before, so after  $t$  years its value is given by the function  $V(t) = 1400 \cdot 0.85^t$ . When will the value of the car jack be \$400?

8. (12pts) The town of Renewal had 34,000 inhabitants in 2015 and 41,000 in 2020. Assume the population of Renewal grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Renewal  $t$  years after 2015. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 50,000?